Emergence of a Partially Split Flux Tube into the Solar Atmosphere

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Abstract

We performed a three-dimensional MHD simulation for the emergence of a partially split flux tube into the solar atmosphere, which was focused on the mechanism of activity observed on the Sun, such as the merging of magnetic polarity regions in the photosphere and the formation of multi-flux domains in the corona. The simulation reproduced that a small polarity region associated with a rotational flow merges into a main polarity region, and the main polarity region later develops a rotational flow in the same direction as the small polarity region. In accordance with the photospheric merging process, multi-flux domains form in the corona and a current layer separating these flux domains travels outward as the flux domains expand. We also investigated the time variations of the magnetic flux, energy, and helicity injected into the atmosphere by the partially split flux tube to see the nature of helicity injection in a multi-pole system.

Key words: methods: numerical — MHD — Sun: corona — Sun: magnetic fields — Sun: photosphere

1. Introduction

The Sun shows various active phenomena on its surface, and the magnetic field has been believed to be a key player in producing these phenomena. The magnetic field travels through the solar interior toward the surface, and after emerging into the surface it expands widely in the solar atmosphere, providing the seeds of activity observed on the Sun. This indicates that active phenomena observed on the Sun are, more or less, related to invisible subsurface processes of the magnetic field, so taking these subsurface processes into account is a key to a good understanding of the physical mechanism of observed phenomena. For example, the so-called flux cancellation is a typical phenomenon observed at the surface, in which the opposite-polarity regions approach each other and disappear at the neutral line between them (Martin et al. 1985; Livi et al. 1985; Li et al. 2004). Observationally, it has been inferred that various kinds of activity are related to flux cancellation: solar flares (Livi et al. 1989; Wang & Shi 1993), microflares/surges/jets (Chae et al. 1999, 2002; Liu & Kurokawa 2004), X-ray bright points (Webb et al. 1993), Ellerman bombs (Pariat et al. 2004), filament formation (Martin 1986), and its activation (Wang et al. 1996; Kim et al. 2001). Flux cancellation even relates to one of the most large-scale events on the Sun, such as coronal mass ejections (CMEs) (Zhang et al. 2001); a model of CMEs based on flux cancellation has been studied (Forbes & Isenberg 1991; Linker et al. 2003).

Flux cancellation might be a direct manifestation of annihilating the magnetic field right on the surface; however if we consider the subsurface process related to this surface phenomenon, the other two mechanisms emerge as possible candidates causing flux cancellation. One of them is the emergence of U-loops (van Driel-Gesztelyi et al. 2000; Bernasconi et al. 2002; Lites 2005), and the other is the submergence of Ω-loops. A magneto-hydrodynamic (MHD) simulation based on the scenario of U-loop emergence has successfully reproduced flux cancellation (Magara 2007).

The present paper deals with a similar phenomenon observed at the surface, in which the same-polarity regions merge together to form a large polarity region (Vrabec 1974; McIntosh 1981; Zwaan 1985). Zhang (1994) reported that the merging of the same-polarity regions is related to the occurrence of major X-class flares, and MHD calculations support this result (Gerrard et al. 2003). Observations have also shown the rotation of a sunspot, into which nearby pores are merging (Brown et al. 2003). When we consider subsurface processes, the apparent merging of polarity regions observed at the surface might be explained by the so-called oak tree model of flux convergence (illustrated by Zwaan 1985, in figure 1). In this model a partially split flux tube continuously emerges into the surface, and when the coalescent part of the tube rises up to the surface, we then observe the merging of individual poles with the same polarity. Beyond cartoon-based considerations, we here aim to clarify the dynamic nature of this merging process. Toward this end, we use a relatively simple geometry of the magnetic field to perform an MHD simulation for the emergence of a partially split flux tube into the solar atmosphere. This study is focused on the flux emergence that connects invisible subsurface processes and observed surface phenomena.

So far, MHD simulations that have focused on flux emergence have successfully explained the physical mechanism of various phenomena observed on the Sun: the dynamic expansion of arch filament systems (Shibata et al. 1989), the formation of sigmoidal loops (Matsumoto et al. 1998; Magara 2004), photospheric shear flows (Manchester 2001; Fan 2001), and their effect on the eruption of a magnetic flux rope into the corona (Manchester et al. 2004; Manchester 2007), and the interaction between the coronal field and the emerging field (Yokoyama & Shibata 1996; Isobe et al. 2005; Archontis et al. 2004, 2007; Galsgaard et al. 2007; Moreno-Insertis et al. 2008). The preemerged condition of the magnetic field...
The physical quantities are gas pressure ($P_g$), density ($\rho$), temperature ($T$), and magnetic pressure ($P_m$). These quantities are scaled to the photospheric values at $z = 0$ ($P_p = 2 \times 10^4$ Pa, $\rho_p = 2 \times 10^{-4}$ kg m$^{-3}$, and $T_p = 6000$ K), and the length is given in Mm.

(c) Same as (b) but for the distribution along the vertical line ($x = -1.4$, $y = 0$) that penetrates the flux tube II.

Below the surface is an important factor in flux emergence, and an extended survey of this condition was conducted by Murray et al. (2006) and Murray and Hood (2007). Very recently, studies incorporating the radiation, thermal conduction, viscosity, and partial ionization into flux emergence have succeeded to make a realistic model of solar atmospheric phenomena (Leake & Arber 2006; Cheung et al. 2007; Abbett 2007; Hansteen et al. 2007).

The organization of this paper is as follows. The next section describes the basic equations used for this study, and explains a numerical simulation that reproduces the emergence of a partially split flux tube. Section 3 presents the results from the MHD simulation, showing the merging of photospheric polarity regions and the formation of multi-flux domains in the corona. In section 4 we discuss a new aspect of flux emergence obtained by the emergence of a partially split flux tube.

2. Description of a Numerical Simulation

2.1. Basic Equations

The numerical simulation performed in this study is based on a set of resistive MHD equations, which are expressed by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla P + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} + \rho \mathbf{g},$$

$$\frac{\rho^{\gamma}}{\gamma - 1} \left[ \frac{\partial}{\partial t} \left( \frac{P}{\rho^\gamma} \right) + (\mathbf{v} \cdot \nabla) \left( \frac{P}{\rho^\gamma} \right) \right] = \frac{\eta}{4\pi} |\nabla \times \mathbf{B}|^2,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}),$$

and

$$P = \frac{\rho RT}{\mu},$$

where $\rho$, $\mathbf{v}$, $\mathbf{B}$, $\gamma$, $\eta$, $\mu$, $R$, and $T$ indicate the gas density, fluid velocity, magnetic field, gas pressure, gravitational acceleration, adiabatic index ($\gamma = 5/3$ is assumed), magnetic resistivity, mean molecular weight ($\mu = 0.5$ is assumed), gas constant, and temperature, respectively. These differential equations were solved using a modified Lax–Wendroff scheme (Magara 1998) in Cartesian coordinates ($x, y, z$) with the $z$-axis directed upward.

2.2. Initial Condition

In order to solve the MHD equations above, we first set up the initial condition in which two isolated flux tubes are surrounded by the hydrostatic gas layers stratified under a uniform gravity (figure 1). These gas layers are characterized by the temperature profile presented in this figure. The solar surface, or the photosphere, is located at $z = 0$. The subsurface layer is slightly superadiabatic, so the layer is unstable against convective motions (Magara 2006). The two flux tubes placed...
below the surface are set parallel to the y-axis, which keep a force-free state inside the tubes and a total pressure balance at their boundaries, so that the tubes are in mechanical equilibrium with the background gas layers. The distribution of the magnetic field in these flux tubes is given by

\[
B = B_0 \frac{\hat{r} + b(z-z_0)\hat{x} - b(x-x_0)\hat{z}}{1 + b^2[(x-x_0)^2 + (z-z_0)^2]} \tag{6}
\]

for \((x-x_0)^2 + (z-z_0)^2 \leq r_f^2\), where \(B_0\) is the field strength at the axis of the flux tube, \((x_0, z_0)\) is the position of the axis in the \((x, z)\)-plane, \(r_f\) is the radius, and \(b > 0\) is a parameter characterizing the degree of a left-handed twist. For the thicker flux tube (Flux tube I), we took \(r_f = 1\) Mm, \(B_0 = 6500\) G, \((x_0, z_0) = (0, -2.6)\) Mm, and \(b = 1.5\), while we took \(r_f = 0.8\) Mm, \(B_0 = 3000\) G, \((x_0, z_0) = (-1.4, -1.2)\) Mm, and \(b = 1.5\) for the thinner flux tube (Flux tube II).

The simulation domain was \((-40, -45, -8)\) Mm \(\leq (x, y, z) \leq (40, 45, 60)\) Mm, which was divided into a number of small cells \((\Delta x, \Delta y, \Delta z)\), which changed size depending on their positions. The smallest cell \((\Delta x, \Delta y, \Delta z) = (0.1, 0.2, 0.1)\) Mm was distributed in the region of \((-5, -15, -8)\) Mm \(\leq (x, y, z) \leq (5, 15, 7)\) Mm. The size of cells started to increase apart from this region, reached \((\Delta x, \Delta y, \Delta z) = (1, 1.2)\) Mm in the largest cell. The total number of cells was then \(N_x \times N_y \times N_z = 233 \times 247 \times 222\). We imposed a rigid boundary condition at the bottom boundary, whereas a free boundary condition was imposed at the other boundaries, and a wave-damping zone was placed in the vicinity of all the boundaries. In this work we used \(77\) s, Mm, \(13\) km s\(^{-1}\) (photospheric sound speed), and \(580\) G as the normalization units of the time, length, velocity, and magnetic field.

2.3. Formation of a Partially Split Flux Tube

The main target of this study was to investigate the emergence of a partially split flux tube, so we needed a process by which two isolated flux tubes could be converted into a single, but partially split flux tube. For this purpose we locally enhanced the magnetic resistivity between the flux tubes to merge them. The following time-series of events describe how to form a partially split flux tube:

For \(0 < t < 0.5\), we locally imposed an upward motion to the flux tube II, which is given by

\[
v_z = v_0 \left[1 + \cos \left(\frac{\pi y}{\lambda}\right)\right] \sin \left(\frac{\pi t}{2t_0}\right) \tag{7}\]

for \((x + 1.4)^2 + (z + 1.2)^2 \leq 0.8^2, -\lambda < y < \lambda, \) where \(v_0 = 0.15, t_0 = 0.5, \lambda = 3.6\).

For \(5 < t < 5.5\), we locally imposed an upward motion to the flux tube I, which is given by

\[
v_z = v_0 \left[1 + \cos \left(\frac{\pi y}{\lambda}\right)\right] \sin \left(\frac{\pi (t - 5)}{2t_0}\right) \tag{8}\]

for \(x^2 + (z + 2.6)^2 \leq 1^2, -\lambda < y < \lambda, \) where \(v_0 = 0.15, t_0 = 0.5, \lambda = 5\).

For \(5 < t < 14\), below the surface \((z < 0)\) we imposed a finite value of magnetic resistivity, \(\eta = 0.01\), on the region of \(j_y = (V \times B)_y < -1\), which helps the two flux tubes to merge at the interface between them. Figure 2 shows a snapshot of the merging process taken at \(t = 7\), in which a partially split flux tube just started to emerge into the surface. After \(t = 14\), we assumed ideal MHD evolution.

3. Results

In this section we present the results, showing various aspects of the activity produced by the partially split flux tube emerging into the solar atmosphere. We start with photospheric activity, such as the merging of the same-polarity regions and the cancellation of the opposite-polarity regions. We then look at a travelling coronal current layer formed in multi-flux domains. We also show the time variations of the quantities injected into the corona, such as the magnetic flux, magnetic energy and magnetic helicity to investigate helicity injection in a multi-pole system. Finally, we explain the configuration of the photospheric magnetic field, by showing how the strength and elevation angle of the photospheric field vary with time and space in an emerging flux region.

3.1. Merging of Photospheric Polarity Regions

Figure 3 shows the temporal development of photospheric magnetic flux when the partially split flux tube emerges into the photosphere. In the early phase \((t = 8)\) when a portion of the flux tube II emerges, a bipolar region appears (the situation here is just after the state presented in figure 2). Then, at \(t = 14\) when a portion of the flux tube I emerges, a new bipolar structure forms around \((x, y) = (0, 0)\), just on the right side of the preemerged bipolar region. The appearance of the newly emerging bipolar region suggests that multi-flux domains form in the corona, which is explained in subsection 3.3 using a field-line map focused on the multi-flux domains. As emergence proceeds, the two bipoles start to merge, which is associated with the emergence of coalesced flux tubes. At \(t = 26\), the photospheric flux shows a distribution that is similar to that produced by a split-less flux tube (e.g., Fan 2001; Magara & Longcope 2003; Archontis et al. 2004).

We then see photospheric plasma motions during the merging process. Figure 4 shows the temporal development of two positive polarity regions merging together. The arrows and
Fig. 3. Temporal development of photospheric magnetic flux. The units of length and magnetic field are given by Mm and 580G.
Fig. 4. Temporal development of photospheric magnetic flux and velocity. The vertical magnetic flux ($B_z$) and horizontal velocity field ($v_x$, $v_y$) are represented by contours and arrows, while the vertical motions ($v_z$) are shown by blue and red color maps. The units of length, magnetic field and velocity are given by Mm, 580G, and 13 km s$^{-1}$ (photospheric sound speed).
colors represent the horizontal ($v_x$, $v_y$) and vertical motions ($v_z$), while the contours indicate the vertical magnetic flux ($B_z$). Polarity regions with intense flux are formed at the footpoints of emerging $\Omega$-loops, and these regions globally translate in the positive $y$-direction while merging. This is caused by the continuous emergence of the $\Omega$-loops, whose footpoints are separating as emergence goes on. Strong downflows are associated with the merging polarity regions.

We further investigate the merging process by decomposing the velocity field around the merging polarity regions into the translation, rotation, expansion/contraction, and distortion. This decomposition is known as the Stokes theorem (Shu 1992), expressed by

$$v = v_{\text{tr}} + \Omega \times R + KR + \nabla_R F,$$

where $v_{\text{tr}}$ shows the translation of the peak flux location (PFL) in a polarity region, and the second, third, and fourth terms on the right-hand side represent the rotation, expansion/contraction, and distortion, respectively. These terms are given by

$$\Omega = \frac{\nabla \times v}{2},$$

$$K = \frac{\nabla \cdot v}{3},$$

$$F = \frac{D_{ij} R_i R_j}{4},$$

where $R = (R_x, R_y, 0)$ is a position vector starting from the PFL and $D_{ij}$ is the deformation rate tensor. The result is shown in figure 5, where two PFLs, PFL I, and PFL II, the former of which represents the flux tube I and the latter the flux tube II, are traced in the central panel. The translation component gives the actual velocity measured at a PFL, which is generally different from the apparent velocity derived by tracing the PFL. Having this in mind, we explain several features of plasma motions during the merging process. To make them clear, we defined two phases characterizing the merging process [the early ($t = 21, 22, 23$) and late phases ($t = 24, 25, 26$)]. When we see the translation component, it clearly shows the global drift of both PFLs in the positive $y$-direction during the early and late phases, as mentioned in figure 4. This component further indicates that these PFLs tend to approach together during the early phase, while in the late phase they stop this tendency, while keeping an interval between them. Looking at the other velocity components, it is found that PFL II is surrounded by a strong rotational flow in the counterclockwise direction during the early phase. This is because flux tube II emerges earlier than flux tube I, so the loop formed by flux tube II is already vertical on the photosphere at this time, driving rotational motions at the photospheric footpoint (Magara 2006). Furthermore, PFL II is subject to strong distortion at $t = 23$, just before a transition from the early to late phase. After the transition is completed, the vicinity of PFL II shows a uniform motion during the late phase (translation is dominant compared to other velocity components). On the other hand, PFL I shows a different behavior compared to PFL II; that is, the vicinity of PFL I takes a uniform motion during the early phase, while a counterclockwise rotational flow becomes prominent in the late phase ($t = 26$). The loop formed by the flux tube I becomes vertical on the photosphere in the late phase, producing a bipolar structure associated with the counterclockwise rotational flow.

### 3.2. Cancellation of Photospheric Flux

The present simulation reproduced the cancellation of magnetic flux in the photosphere, and we explain this by selecting two different events. The first event is observed in a relatively early phase when the negative pole is pushed toward the positive pole in the preemerged bipole by the newly emerging bipole. Figure 6a shows a snapshot of this event taken at $t = 18$, in which the contours represent the vertical magnetic flux while the arrows indicate the horizontal velocity. The black dashed line is the neutral line and the area outlined with the pink box is a focused region where the time variation of the vertical flux has been calculated. A field line is drawn by the blue line. The field line has the shape of an U-loop, as indicated by figure 6b, in which the velocity field on this field line (red arrows) is displayed. The time variation of the vertical flux is presented in figure 6c after the flux transported across the boundary of the pink box has been corrected, and this figure shows the net decrease of positive and negative fluxes. Figure 6d shows the property of the U-loop presented in figure 6b, such as the shape and motion, where the loop length has been measured from the point ‘A’ indicated in figures 6a and 6b. The field-aligned velocity ($V_z$) in figure 6d shows a diverging flow pattern around the dip of the loop. The positive value of the vertical velocity ($V_z$) at the dip indicates that the U-loop is emerging.

Another event was selected at the late time when the flux tube I entered a developed phase of emergence. Figure 7a shows a snapshot taken at $t = 24$, and at this time the merging of photospheric polarity regions almost completed. In this event, a focused region is located in the area where a global diverging flow is observed. Figures 7b–7d indicate that U-loop emergence operates to cause a cancellation of the flux, as we just described based on figures 6b–6d.

### 3.3. Travelling Current Layer in the Corona

After emerging, the partially split flux tube forms multi-flux domains in the corona. This is displayed in figure 8, where field lines composing each flux domain are drawn in different colors. The blue field lines occupy the flux domain produced by the flux tube II that emerged first, while the red field lines represent the field lines of the flux tube I that emerged late. Even after the photospheric polarity regions merged together to form a single bipolar-like structure, the coronal flux domains are preserved and a current layer forms at the interface between the flux domains. Figure 9 shows a time-series of two-dimensional viewgraphs of the evolving flux domains and current layer in a vertical plane sectioning the flux domains at $y = 0$. The arrows and grayscale map represent the velocity field projected on this plane ($v_x, v_z$) and $\log j$, where $j = |\nabla \times B|$. The contours show the plasma beta in logarithmic scale. Since the magnetic field is generally weak in the current layer, the plasma beta becomes large there. Figure 9 shows that the flux domains expand and the current layer travels outward in the corona.
Fig. 5. Temporal development of the decomposed photospheric velocity field during the merging process. The red, blue, green, and black arrows represent the components of distortion, rotation, expansion/contraction, and translation, respectively. The diamonds in the central panel show the position of two PFLs (PFL I and II represent the flux tubes I and II, respectively). Each PFL is traced by a dashed line. The units of length and velocity are given by Mm and 13 km s$^{-1}$ (photospheric sound speed).
Fig. 6. (a) Photospheric distributions of vertical magnetic flux (contours) and horizontal velocity field (arrows) obtained at $t = 18$. The area outlined with the pink box is where the magnetic flux was calculated and the blue line passing this area represent a field line. The black dashed line indicates a neutral line. The units of length, magnetic field, and velocity are given by Mm, 580 G, and $13 \text{ km s}^{-1}$ (photospheric sound speed). (b) Perspective viewgraph of the field line presented in figure 6a and the velocity field on this field line (arrows). (c) Time variation of the magnetic flux calculated in the area outlined with the pink box in figure 6a. Dotted and solid lines represent the negative and positive fluxes, respectively. The flux is normalized by $5.8 \times 10^{18} \text{ Mx}$. (d) Distributions of the height and velocity components along the field line shown in figures 6a and 6b. The black, red, green, and blue lines represent the height, $v_x$, $v_y$, and $v_z$, respectively. The pink line indicates field-aligned velocity. The length of the field line is measured from the footpoint represented by 'A' in figures 6a and 6b. The units of length and velocity are given by Mm and $13 \text{ km s}^{-1}$ (photospheric sound speed).
Fig. 7. Same as figures 6a–6d, except for $t = 24$. 
Fig. 8. Distribution of emerging field lines and their temporal development. Blue and red field lines are distributed in the flux domains formed by the flux tubes II and I, respectively. The grayscale map at the base shows photospheric magnetic flux. The units of length and magnetic field are given by Mm and 580 G.
Fig. 9. Distributions of the strength of current density ($|\nabla \times B|$, grayscale map), plasma beta (contours), and velocity field (arrows) projected onto the $y = 0$ plane, obtained at four different times. The strength of current density and the plasma beta are displayed in logarithmic scale. The units of length and velocity are given by Mm and $13\text{km s}^{-1}$ (photospheric sound speed).
3.4. Magnetic Quantities Injected into the Atmosphere

We then consider the physical quantities injected into the atmosphere by the partially split flux tube. Firstly, we calculated the time variations of the magnetic flux, magnetic energy, and magnetic helicity stored in the atmosphere \((z > 0)\). The magnetic flux was integrated over the surface \((z = 0)\) and a vertical plane \((y = 0)\) as follows:

\[
\Phi_+ = \int_{z=0} B_z \, dx \, dy \quad \text{for } B_z > 0, \quad (13)
\]
\[
\Phi_- = \int_{z=0} B_z \, dx \, dy \quad \text{for } B_z < 0, \quad (14)
\]

and

\[
\Phi_y = \int_{y=0, z \geq 0} B_y \, dx \, dz. \quad (15)
\]

The magnetic energy and magnetic helicity were calculated by

\[
E_m = \int_{z \geq 0} \frac{B^2}{8\pi} \, dx \, dy \, dz, \quad (16)
\]

and

\[
H_m = \int_{z \geq 0} A \cdot B \, dx \, dy \, dz, \quad (17)
\]

respectively. \(A\) is a vector potential for \(B\), related by \(B = \nabla \times A\). To make \(H_m\) gauge-invariant in a volume bounded by the surface, we used the relative helicity, giving the difference from the potential field that has the same flux distribution at the boundary (Berger & Field 1984; Finn & Antonsen 1985). In this case, the vector potential is given by (DeVore 2000)

\[
A(x, y, z, t) = A_C(x, y, 0, t) - \hat{z} \times \int_0^z B(x, y, z', t) \, dz'. \quad (18)
\]

Here, \(A_C\) is a vector potential for the potential field that has the same photospheric distribution of vertical magnetic flux as \(B\), and given by

\[
A_C(x, y, z, t) = \nabla \times \hat{z} \int_z^\infty \phi_C(x, y, z', t) \, dz'. \quad (19)
\]

\(\phi_C\) is a scalar potential for that potential field, and given by

\[
\phi_C(x, y, z, t) = \frac{1}{2\pi} \int_{z=0} B_z(x', y', 0, t) \, dx' \, dy'. \quad (20)
\]

Figures 10a–10c display the time variations of the quantities \(\Phi_+\), \(\Phi_-\), \(\Phi_y\), \(E_m\), and \(H_m\), indicating that there are two phases during which these quantities are efficiently injected into the atmosphere. The first phase is \(3 < t < 7\), followed by the second phase of \(15 < t\). Efficient injection is associated with the emergence of flux tube II during the first phase and flux tube I during the second phase. A point to be noticed here is the time variation of the magnetic flux. It shows that \(\Phi_+\) and \(\Phi_-\) take almost the same absolute value, indicating that the net flux across the photosphere keeps zero throughout the simulation. On the other hand, the absolute value of \(\Phi_y\) is always smaller than that of \(\Phi_+\) and \(\Phi_-\). This discrepancy is caused by the field lines penetrating the surface multiple times due to their helical structures, which apparently increases the signed magnetic flux obtained at the surface.

Next, we investigated the Poynting flux and helicity flux obtained at the surface. These quantities are given by

\[
F_{Mz} = -\frac{1}{4\pi} \int_{z=0} (B_x v_x + B_y v_y) B_z \, dx \, dy + \frac{1}{4\pi} \int_{z=0} (B_x^2 + B_y^2) v_z \, dx \, dy \quad (21)
\]
\[ F_{Hz} = -2 \int_{z=0}^{t} (A_xv_x + A_yv_y) B_z \, dx \, dy \]
\[ + 2 \int_{z=0}^{t} (A_xB_x + A_yB_y) v_z \, dx \, dy. \]  

The first term on the right-hand side of equations (21) and (22) represents the work done by horizontal motions, and will be called the shear term. The second term, called the emergence term, would clearly vanish in the absence of vertical flows. The time variations of \( F_{Mz} \) and \( F_{Hz} \) are presented in figure 10d and 10e, which also show that there are two phases of the efficient injection of the magnetic energy and magnetic helicity. Besides that, we found several features of flux emergence during an intermediate phase between the phases of efficient injection. Figure 10d shows that the emergence term is negative during the intermediate phase, which is caused by strong downflows developing at the photospheric footpoints of the flux tube II, reducing the magnetic energy in the atmosphere. These downflows are a typical phenomenon observed when an emerging flux tube enters a developed phase and the footpoints become vertical on the photosphere (Magara & Longcope 2003). Figure 10e shows that the flux of the magnetic helicity becomes positive in the emergence term during the intermediate phase (11 < \( t < 15 \)). This is quite different from the case of a split-less flux tube, in which negative helicity is predominantly injected via emergence into the atmosphere when the flux tube has a left-handed twist. The distribution of photospheric helicity flux is displayed in figure 11, indicating that negative helicity flux is dominant at all times, except for \( t = 14 \), at which positive helicity flux dominates in the region where the newly emerging bipolar region appears on the right side of the preemerged bipolar region.

The positive helicity flux found during the intermediate phase is explained in terms of the mutual helicity that comes into play in a multi-pole system. We here estimate this in a qualitative manner. Figure 12a illustrates the relative position of two bipoles, and a set of values \( a, b, d, \alpha, \beta \) are specified to determine the position of four poles: \( P1, P2 \) (positive), \( N1, N2 \) (negative). The mutual helicity in this configuration is given by

\[ H = \frac{\theta_1 - \theta_2}{\pi} F_{P1-N1} F_{P2-N2}, \]  

where \( \theta_1, \theta_2 \) are the angles displayed in figure 12b and \( F_{P1-N1}, F_{P2-N2} \) represent the magnetic flux contained by the bipole \( P1-N1 \) and \( P2-N2 \), respectively (Berger 1998; Priest & Forbes 2000). For example, we derived \( (a, b, d, \alpha, \beta) = (4.01, 0.955, 1.81, 0.286, 0.933) \) from the \( t = 14 \) map in figure 11, assuming that the poles are located at the PFL in each polarity region. In this case, equation (23) gives

\[ H = 0.174 F_{P1-N1} F_{P2-N2}. \]  

so we have a positive value of the mutual helicity. Figure 12c presents a result of modeling the evolution of the bipole P2–N2, from its appearance to the subsequent clockwise rotation leading to the state at \( t = 14 \) (\( b \) and \( \beta \) are given by linear functions of a normalized time). This figure shows that the evolution produces the positive mutual helicity that increases with time (\( F_{P2-N2} \) temporally increases during emergence).

This is consistent with the positive helicity flux observed at \( t = 14 \) in figure 11.

3.5. Configuration of Photospheric Magnetic Field

The configuration of the magnetic field distributed in the photosphere is an important factor of the activity observed on the Sun, and recent observations with advanced instruments have revealed in detail the dynamic and inhomogeneous nature of the photospheric magnetic field (Kubo et al. 2003; Ishikawa et al. 2008; Lites et al. 2007). Figure 13 shows the temporal development of the strength (left panels) and elevation angle (middle panels) of the photospheric magnetic field distributed in an emerging flux region. The right panels show histograms of the number of grid points, which depends on the field strength and elevation angle observed at these points. To make these histograms, we firstly distributed a grid point with an interval of 0.5 in the \((x, y)\)-domains, and then counted the number of grid points when the field strength and elevation angle fall within a certain range of values. These histograms show that a weak-field region is dominated by horizontal fields (elevation angle is close to zero) all the time, while a strong-field region is dominated by horizontal fields in the early phase of emergence, although strong fields become vertical and tend to be confined to a small area, forming a pole in the late phase. The region where the field strength is less than the equipartition value of convective kinetic energy (about 500 G, which corresponds to \( b \approx 1 \) in figure 13) could be disturbed by convective motions, so the configuration of the magnetic field in such a region could be significantly deformed when vigorous convective motions operate. In a developed phase (\( t = 26 \)), the large area of the emerging flux region is covered by such a weak-field region where horizontal fields dominate, which is subject to convective motions, and could be disintegrated. Accordingly, a bipolar structure made of strong vertical fields surrounded by a disintegrated horizontal-field region will be a possible configuration observed in a developed emerging flux region.

4. Discussion

The results presented above show that a partially split flux tube produces remarkable activity when emerging into the solar atmosphere. We here discuss a new aspect of flux emergence derived from these results. As mentioned in the introduction, we used a relatively simple geometry of the magnetic field in the present study, and this may cause a significant difference between the simulation results and actual observations. We pick up this point below.

4.1. Merging of Photospheric Polarity Regions

The merging of photospheric polarity regions is one of the important processes caused by the emergence of a partially split flux tube. This converts two separate polarity regions into a single region, so it is a fundamental process causing a significant change in the distribution of the photospheric magnetic field. The merging process itself is also interesting in that a particular flow pattern is found in the merging polarity regions. In this simulation two flux tubes emerged to form polarity regions in the photosphere, and a small polarity region
Fig. 11. Temporal development of photospheric magnetic flux (contours), helicity flux (grayscale map), and horizontal velocity field (red arrows). The units of length, magnetic field, helicity flux, and velocity are given by Mm, 580G, $4.4 \times 10^{19}$ Mx cm$^{-2}$ s$^{-1}$, and 13 km s$^{-1}$ (photospheric sound speed).
4.2. Flux Cancellation and Associated Flows

Another important result on the photospheric magnetic field might be the cancellation of positive and negative fluxes. We reported two events of flux cancellation: one observed in the preemerged bipolar region where the negative pole is pushed toward the positive pole by the newly emerging bipolar region; the other was observed in the newly emerging bipolar region during the late phase when a global diverging flow develops near the canceling region. These two events are schematically explained in figure 14, showing a cross-section viewgraph of the partially split flux tube obtained at $y = 0$. In this figure the flux tubes I (solid line) and II (dotted line) form the newly emerging and preemerged bipolar regions, respectively. In the intermediate phase there appears to be a flow directed toward the neutral line inside the preemerged bipolar region, which is associated with flux cancellation. The origin of this kind of flows, called converging flows when they appear in both sides of the neutral line, is an important issue when we discuss flux cancellation. In the present case the origin is a diverging flow produced by the newly emerging bipolar region that appears near the preemerged bipolar region.

In both events discussed above, U-loop emergence works as a basic mechanism for flux cancellation, although this does not exclude the possible role of magnetic reconnection in producing flux cancellation. Since we assumed ideal MHD evolution from $t = 14$, a reconnection at the photospheric end of the current layer in the intermediate phase was prohibited in the present simulation, which could work as another mechanism for flux cancellation.

4.3. Complex Coronal Field Formed above Simple Photospheric Field

Although the photospheric field tends to form a simple bipolar-like structure in the late phase, the coronal field remains complex, forming multi-flux domains accompanied by a dynamic current layer. The ‘dynamic’ means that this current layer does not stay at the same position; rather, it travels in the corona as the flux domains expand. The state of affairs is significantly different from the case of a split-less flux tube, in which emerging magnetic field forms the global configuration composed of a single flux domain without a travelling current layer. Such a travelling current layer might cause the coronal activity producing energetic particles that enter the layer, whose activation site changes with time as the current layer travels in the corona. The coronal activity is expected to occur above the region where the merging of photospheric polarity regions proceeds. A suggestion here is that if the preexisting field is so tight that the current layer does not travel, rather it stays at the same position to build up free energy by increasing current in the layer, then explosive events, such as flares might occur. This destroys multi-flux domains and reduces a complex coronal field to a simple one, just matching it to the simple bipolar configuration of the
Fig. 13. Temporal development of the strength (left panels) and elevation angle (middle panels) of photospheric magnetic field, where the length scale is given by Mm. The right panels show histograms of the number of grid points, which depends on the field strength and the elevation angle observed at these points. The field strength is normalized by 580G, while the elevation angle is given by radian in the middle panels and degree in the right panels.
photospheric field. The details of the explosive process are now under thorough investigations (Isobe et al. 2005; Archontis et al. 2007; Galsgaard et al. 2007; Moreno-Insertis et al. 2008).

In this simulation, we produced a partially split flux tube by merging two separate flux tubes, although this is just a (technical, in some sense) way to produce it, and there might be other processes operating in the Sun by which a partially split flux tube is formed. For example, strong convective motions could give rise to a single twisted flux with partial fragmentation at its apex portion. If this flux tube emerges into the atmosphere, then a travelling current layer that we reported may not appear, and the subsequent evolution could be completely different from that shown in this study. We definitely need an extended survey of this process, and probably the information provided by observations will make a significant contribution to studying the property of flux emergence full of variety.

In summarizing this work, we investigated the dynamic process caused by a partially split flux tube emerging into the solar atmosphere. To obtain a deep understanding of merging photospheric polarity regions and associated coronal activity, we need to combine the simulation results with advanced observations that have high spatial and temporal resolutions. These observations are even more available now than before by a new satellite mission, Hinode.

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