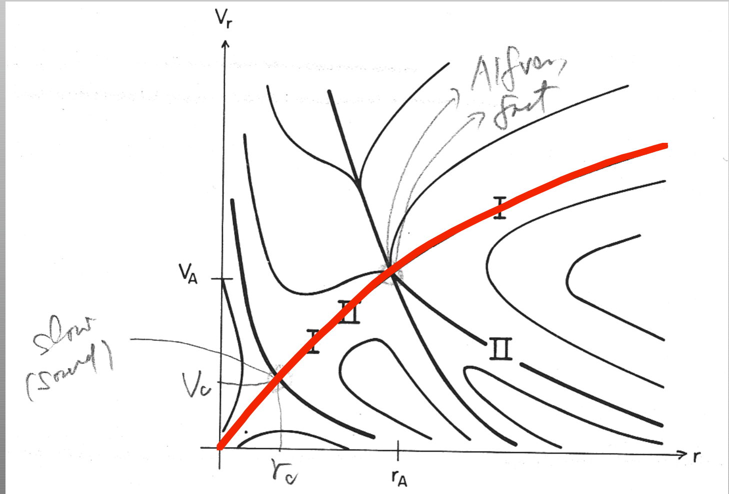
6 integrated equations for 6 quantities ($\rho, v_r, v_\varphi, p, B_r, B_\varphi$)

→ **one equation** for **one quantity** ($v_r(r)$)

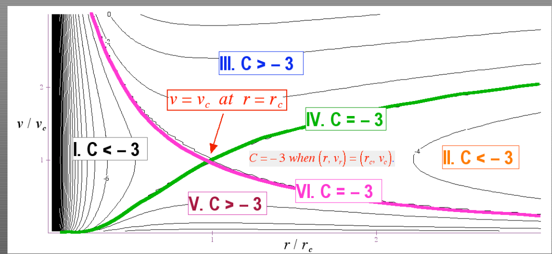
Profile of $v_r(r)$



There are three critical points corresponding to three characteristic wave speeds: **slow-mode speed**, **Alfvén speed**, **fast-mode speed**.

Solution I is appropriate for solar wind.

$r_c \sim 5\ R_\odot\ ,\ r_A \sim 10\ R_\odot$ *with* $T \sim 10^6$ K , $B_{surface} \sim 1$ G



Parker's solution (there is only one critical point corresponding to sound speed)

For details, see http://163.180.179.74/~magara/page31/Topics/SolarWind/sw.html.

Profile of $v_\varphi(r)$

$$v_\varphi B_r - v_r B_\varphi = \Omega r B_r$$

$$r v_\varphi - \frac{B_r}{4\pi\rho v_r} r B_\varphi = L$$

eliminate B_φ

$$v_\varphi = \Omega r \boxed{\frac{\frac{L}{\Omega r^2} M_A^2 - 1}{M_A^2 - 1}}$$

$$M_A \equiv \frac{v_r}{v_A}, \quad v_A \equiv \frac{B_r}{\sqrt{4\pi\rho}}$$

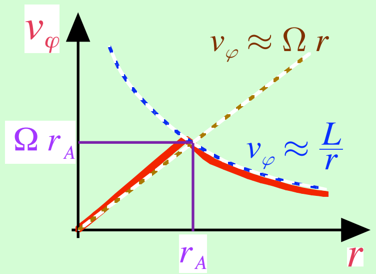
When $M_A = 1$ $\left(r = r_A \ldots \textit{Alfvén critical point}\right)$, the numerator should be zero:

$$\frac{L}{\Omega r^2} M_A^2 - 1 = 0 \Rightarrow L = \Omega r_A^2 = r_A r_A \Omega$$

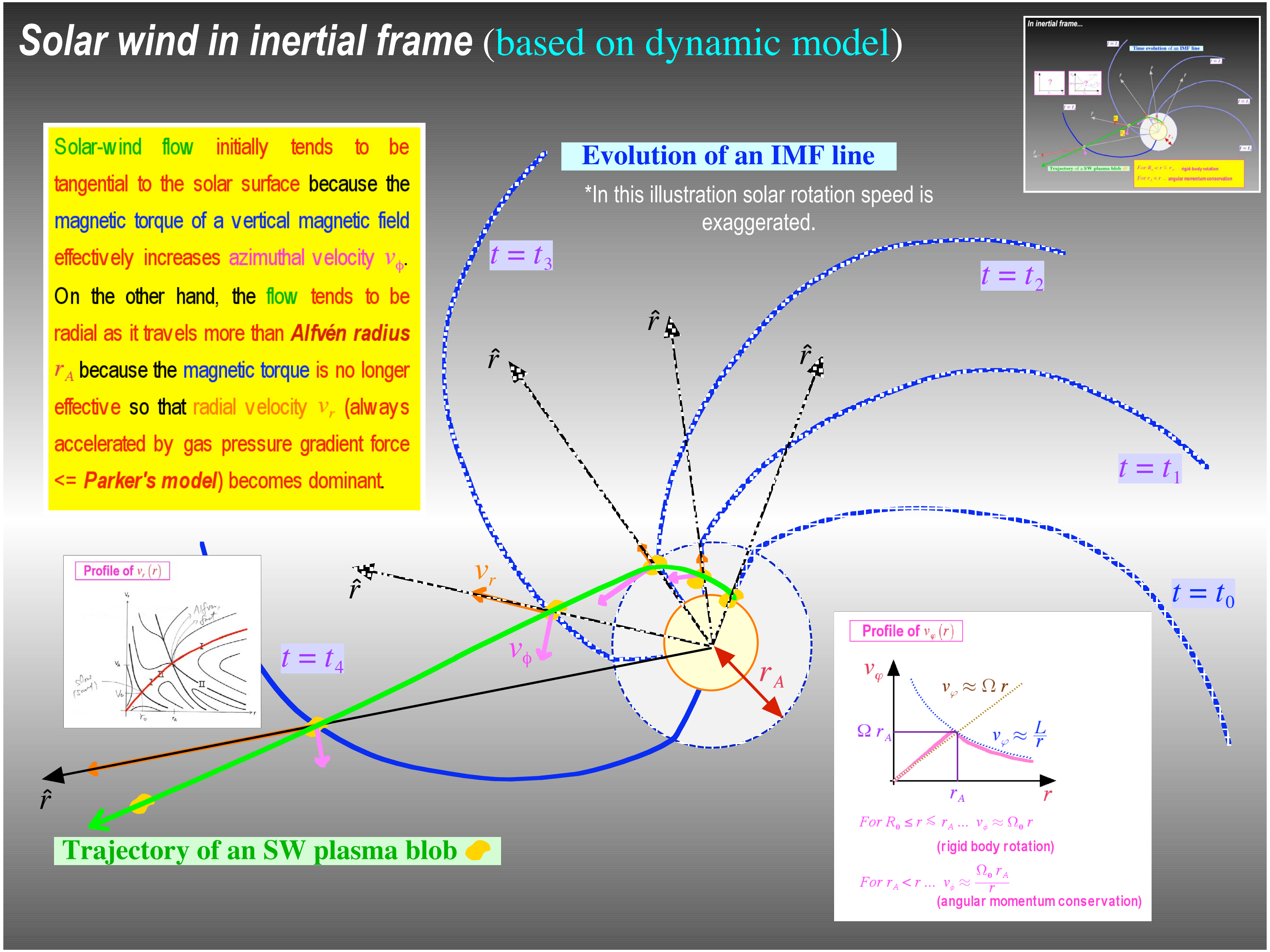
This indicates that the Alfvén critical point rotates at the same rate as the solar equator (**rigid-body rotation**).

$$v_\varphi = \Omega r \frac{\left(\frac{r_A}{r}\right)^2 M_A^2 - 1}{M_A^2 - 1}$$

When $R_\odot \le r < r_A$ => $M_A \ll 1$, $v_\varphi \approx \Omega r$.
(**rigid-body rotation** via strong magnetic torque)

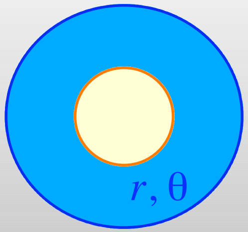


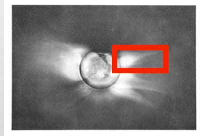
When $r_A < r$ => $M_A \gg 1$, $v_\varphi \approx \frac{L}{r}$.
(angular momentum per unit mass is almost conserved $J = v_\varphi r \sim L = const.$
because magnetic torque is weak, indicating that solar wind plasma with frozen-in IMF is delayed against the solar rotation => **spiral shape of the IMF**)


Solar wind in inertial frame (based on dynamic model)
Solar-wind flow initially tends to be tangential to the solar surface because the magnetic torque of a vertical magnetic field effectively increases azimuthal velocity v_ϕ. On the other hand, the flow tends to be radial as it travels more than Alfvén radius r_A because the magnetic torque is no longer effective so that radial velocity v_r (always accelerated by gas pressure gradient force <= Parker's model) becomes dominant.
Evolution of an IMF line
*In this illustration solar rotation speed is exaggerated.
$t = t_3$
$t = t_2$
$t = t_1$
$t = t_0$
$t = t_4$
$\hat{r}$
v_r
v_ϕ
r_A
Profile of $v_r(r)$
Profile of $v_\varphi(r)$
v_φ
$v_\varphi \approx \Omega r$
Ωr_A
$v_\varphi \approx \frac{L}{r}$
r_A
r
For $R_\odot \le r \lesssim r_A$ ... $v_\varphi \approx \Omega_\odot r$
(rigid body rotation)
For $r_A < r$ ... $v_\varphi \approx \frac{\Omega_\odot r_A}{r}$
(angular momentum conservation)
Trajectory of an SW plasma blob
In inertial frame...

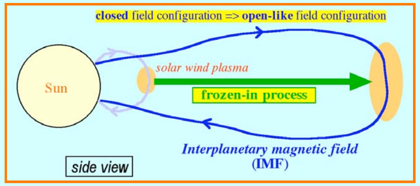
Pneuman-Kopp 2-dimensional model

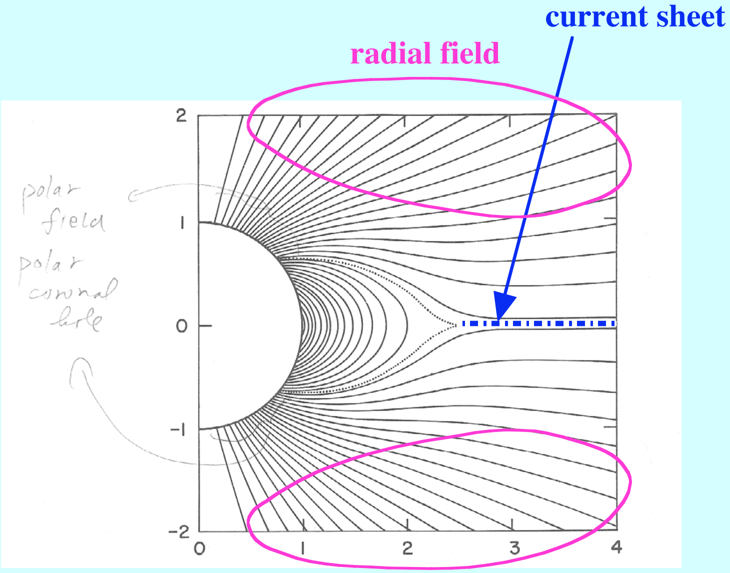
r & θ–components of a vector





(axis-symmetric, on a meridional plane, isothermal, steady, dipole configuration, no rotation)
(depends on *r & θ*, *r & θ–components of a vector* are considered)





Solid lines... **field lines** and **stream lines** (they are parallel because of a steady state)

Basic equations (steady)

$$\nabla \bullet (\rho \boldsymbol{v}) = 0$$

$$\rho\left(\boldsymbol{v} \bullet \nabla \boldsymbol{v}\right) = -\nabla p + \boldsymbol{j} \times \boldsymbol{B} - \frac{G M_\odot \rho}{r^2}\hat{\boldsymbol{r}} = 0$$

$$T = const.$$

$$\nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0$$

Boundary condition (at the solar surface)

$$\rho = \rho_0 \qquad B_r = B_0 \cos\theta \qquad T_0 = 1.56\times10^6\ K$$

$$p_0 = \frac{\rho_0}{\mu} k_B T_0 = \frac{B_0^2}{2\mu_0}$$

Comparison with potential field

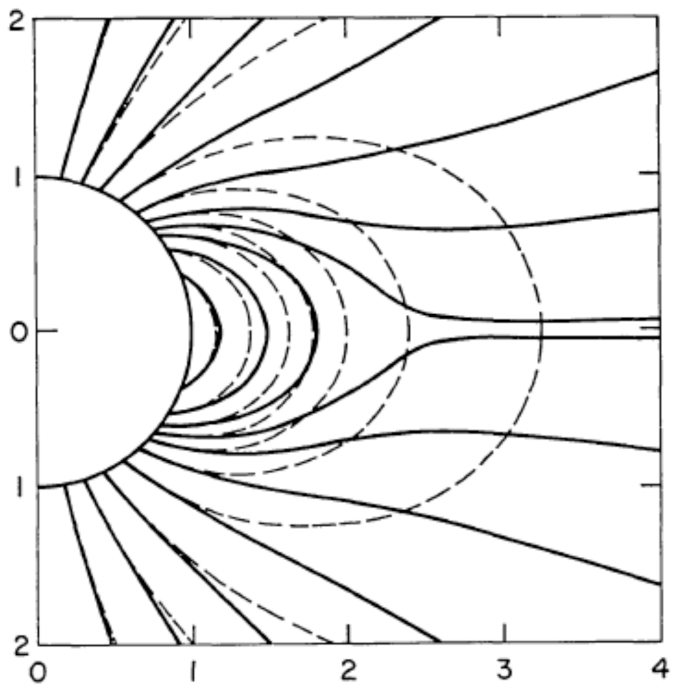
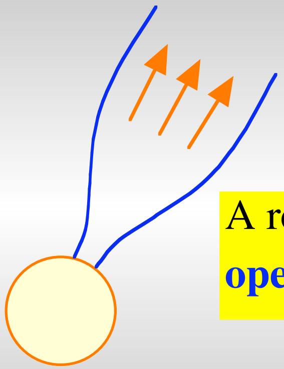


Fig. 8. A comparison of the magnetic field of Figure 3 (solid curves) with a potential field (dashed curves) having the same normal component at the reference level. The field lines for the two configurations are chosen so as to be coincident at the surface.

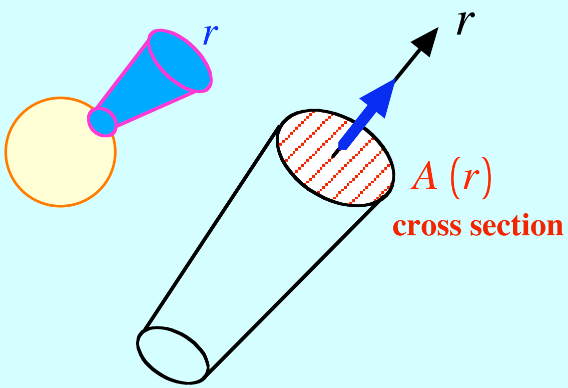
Model of solar wind in a coronal hole

What is a coronal hole?



A region where **solar wind** flows along **open-like magnetic field lines**.

Parker's model is a simple model of solar wind in a coronal hole.



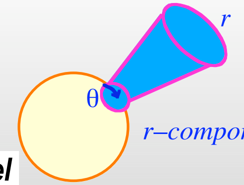
In this model, the cross sectional area is given by

$$A(r) = A_\odot \left(\frac{r}{R_\odot} \right)^2$$

spherically symmetric flux tube

Wang-Sheeley-Arge model

Empirical model of solar wind in a coronal hole

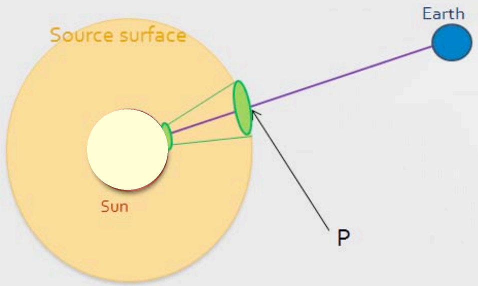


2-dimensional model

(depends on r, θ. *r–component of a vector* is considered)

- The potential-field source-surface method is used to determine the coronal field configuration from the observed photospheric field and to evaluate f_s
- f_s is flux-tube expansion factor. It measures the rate of cross section between the photosphere and the source surface at which the Earth-directed flux tube expands radially.

$$f_s = \left(\frac{R_\odot}{R_S}\right)^2 \left[\frac{B^P(R_\odot)}{B^P(R_S)}\right] \leftrightarrow f_s = \frac{A(r)}{A^{Parker}(r)} \quad A^{Parker}(r) \propto r^2$$



CORRESPONDENCE BETWEEN FLUX-TUBE EXPANSION FACTORS AND DAILY WIND SPEEDS

f_s	v_w (km s^{-1})
< 3.5	> 650
3.5–9	650–550
9–18	550–450
> 18	< 450

Empirical formula for v_r

$$V(f_s,\theta_b) = 265 + \frac{1.5}{(1+f_s)^{1/3}} \left\{5.8 - 1.6 e^{\left[1-\left(\theta_b/7.5\right)^3\right]}\right\}^{3.5} \text{ km s}^{-1}$$

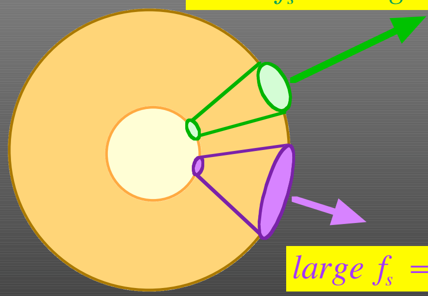
Where:

f_s = Magnetic field expansion factor.

θ_b = Minimum angular distance that an open field footpoint lies from nearest coronal hole boundary (i.e., Angular depth inside a coronal hole)

Solar wind speed vs. flux expansion factor

=> inversely correlated



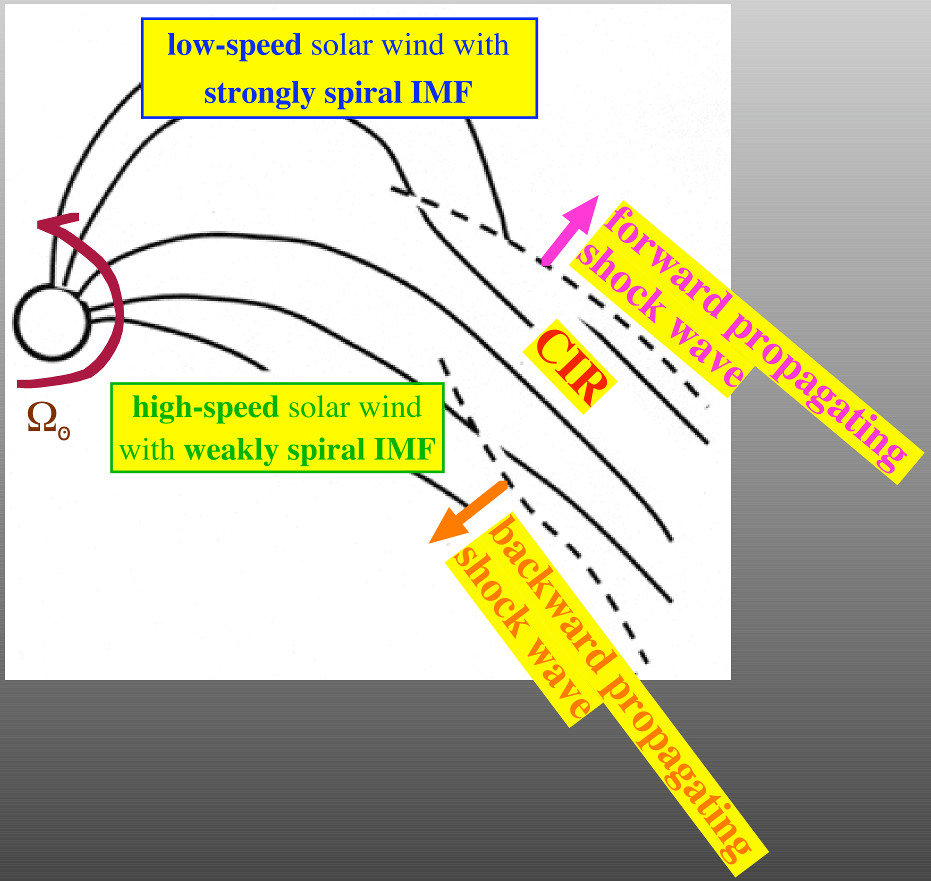
Non-steady phenomena in solar wind

CIR (corotating interaction region)

ICME (interplanetary coronal mass ejection)

CIR (Corotating Interaction Region)

CIR… A region where a *high-speed solar wind* interacts with a *low-speed solar wind* ahead. This is a location where **MHD shock waves** can be formed.



ICME

ICME speed > sound speed, Alfvén speed

↓

MHD shock wave

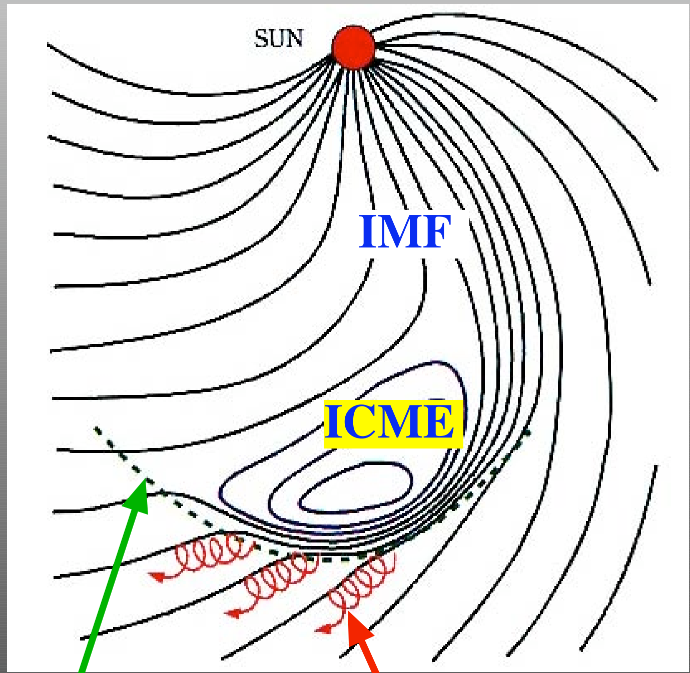
- **radio emission**

  observed as **type II burst (metric range)**

  due to plasma oscillation driven by MHD shock wave

- **particle acceleration**

  via scattering process



Energy ~ 100 Mev => ***relativistic proton*** ($v_p \geq 0.3\ c$)

=> It takes about 20 min. to travel from the corona to the Earth.

=> The high-energy proton can penetrate a barrier of terrestrial magnetic field (***proton event***).

Long-term variation of solar wind

Solar wind variations over multiple solar cycles

positive correlation

$n_{He} \leq 0.1\ n_{proton}$ in the corona

During solar maximum many active regions appear, which increases proton's density in the corona. Hence, more He particles acquire enough momentum to reach an Earth orbit via enhanced collision rate between proton & He.

Sunspot number

positive correlation

IMF field strength

Wind Velocity

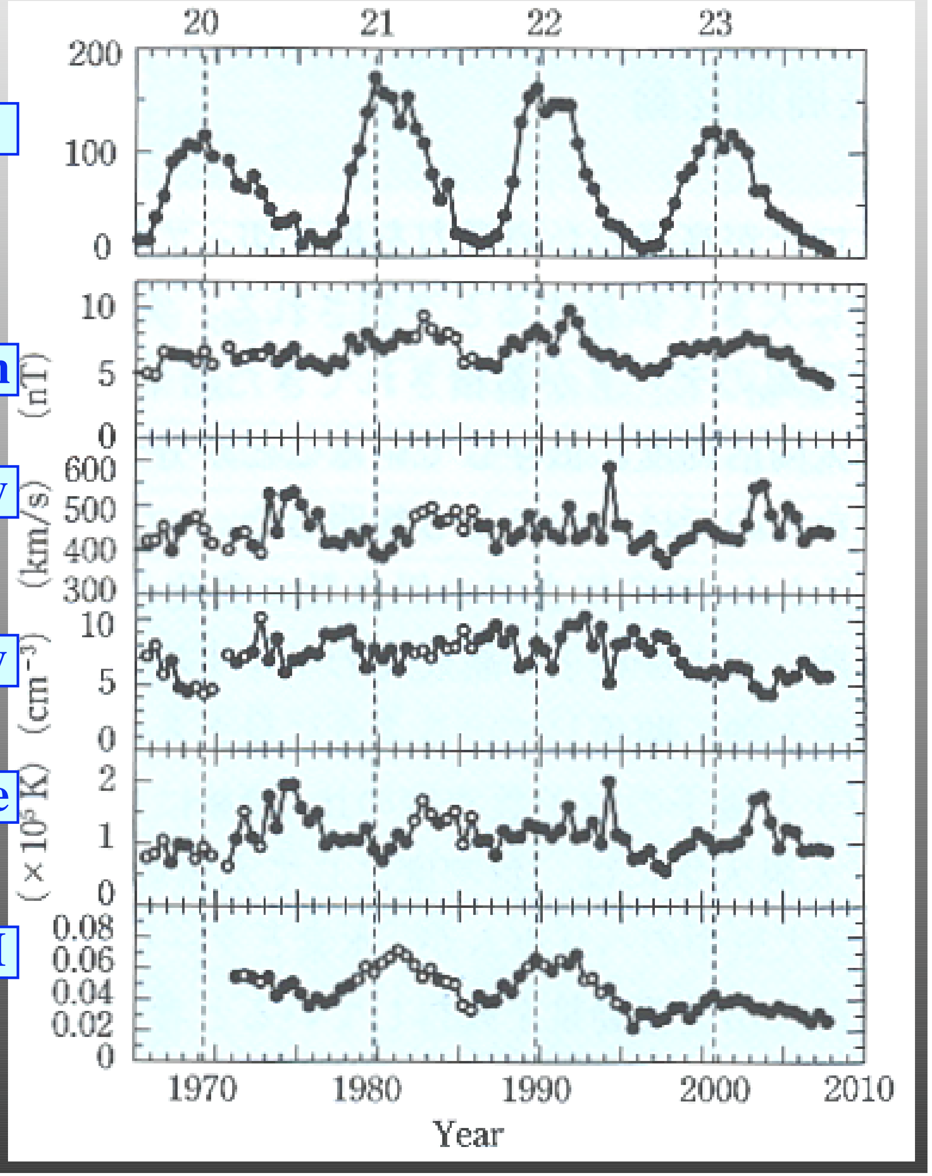
negative correlation

Density

positive correlation

Temperature

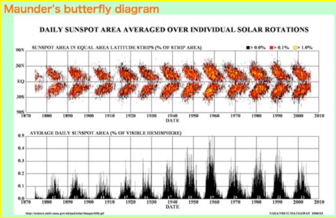
He/H



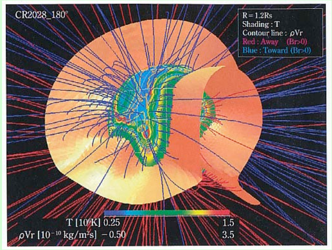
Three key factors determining physical properties of solar winds...

Solar activity

related to **magnetic field strength**



Global magnetic field configuration



Generation mechanism of SW plasma

related to **solar atmospheric heating** & **mass supply** from **the photosphere** to **the corona**