Ohm's law

Subtract ***electron's momentum equation*** *x* ***M*** *from* ***proton's momentum equation*** *x* ***m***

(subtration of $(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}$ terms is neglected <= *large spatial scale, long time scale*)

$$n\,\frac{M\,m}{e}\,\frac{\partial}{\partial t}\left(\frac{\boldsymbol{j}^{MHD}}{n}\right) = e\,\rho^{MHD}\,\boldsymbol{E} + (M+m)\,\boldsymbol{f}_c^{\,p-e} - m\,\nabla P^p + M\,\nabla P^e + n\,e\,\left(m\,\boldsymbol{v}^p + M\,\boldsymbol{v}^e\right)\times\boldsymbol{B}$$

$$\boldsymbol{f}_c^{\,p-e} = -\,n\,e\,\eta\,\boldsymbol{j}^{MHD}$$

$-\boldsymbol{f}_c^{\,p-e} \propto \boldsymbol{j}^{MHD}$ (*both* $\propto \boldsymbol{v}^p - \boldsymbol{v}^e$)

MKS

$$\eta = \frac{M}{n\,e^2\,\tau_c^{p-e}} = \frac{m}{n\,e^2\,\tau_c^{e-p}}$$: resistivity

$$\eta_{diff} \equiv \frac{1}{\mu_0}\,\eta = \frac{c^2\,\tau_p^2}{\tau_c^{e-p}}$$: magnetic diffusivity

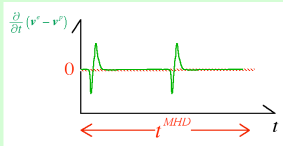
$$m\,\boldsymbol{v}^p + M\,\boldsymbol{v}^e = M\,\boldsymbol{v}^p + m\,\boldsymbol{v}^e + M\left(\boldsymbol{v}^e - \boldsymbol{v}^p\right) + m\left(\boldsymbol{v}^p - \boldsymbol{v}^e\right)$$
$$= \frac{\rho^{MHD}}{n}\,\boldsymbol{v}^{MHD} - (M-m)\,\frac{\boldsymbol{j}^{MHD}}{ne}$$

$$\boldsymbol{E} = -\,\boldsymbol{v}^{MHD}\times\boldsymbol{B} + \eta\,\boldsymbol{j}^{MHD} + \frac{M}{e\,\rho^{MHD}}\left[\frac{n\,m}{e}\,\frac{\partial}{\partial t}\left(\frac{\boldsymbol{j}^{MHD}}{n}\right) + \left(1 - \frac{m}{M}\right)\boldsymbol{j}^{MHD}\times\boldsymbol{B} + \nabla\left(\frac{m}{M}P^p - P^e\right)\right]$$

approximation *assumption* *approximation* *approximation*

assumption & approximation

Slow evolution: $\frac{\partial}{\partial t}\left(\frac{\boldsymbol{j}^{MHD}}{n}\right) \sim 0 \Leftarrow \frac{\partial}{\partial t}\left(\boldsymbol{v}^p - \boldsymbol{v}^e\right) \sim 0$ *on MHD time scale*



Only $\left(\frac{m}{M}\right)^0$ **-term** is taken into account.

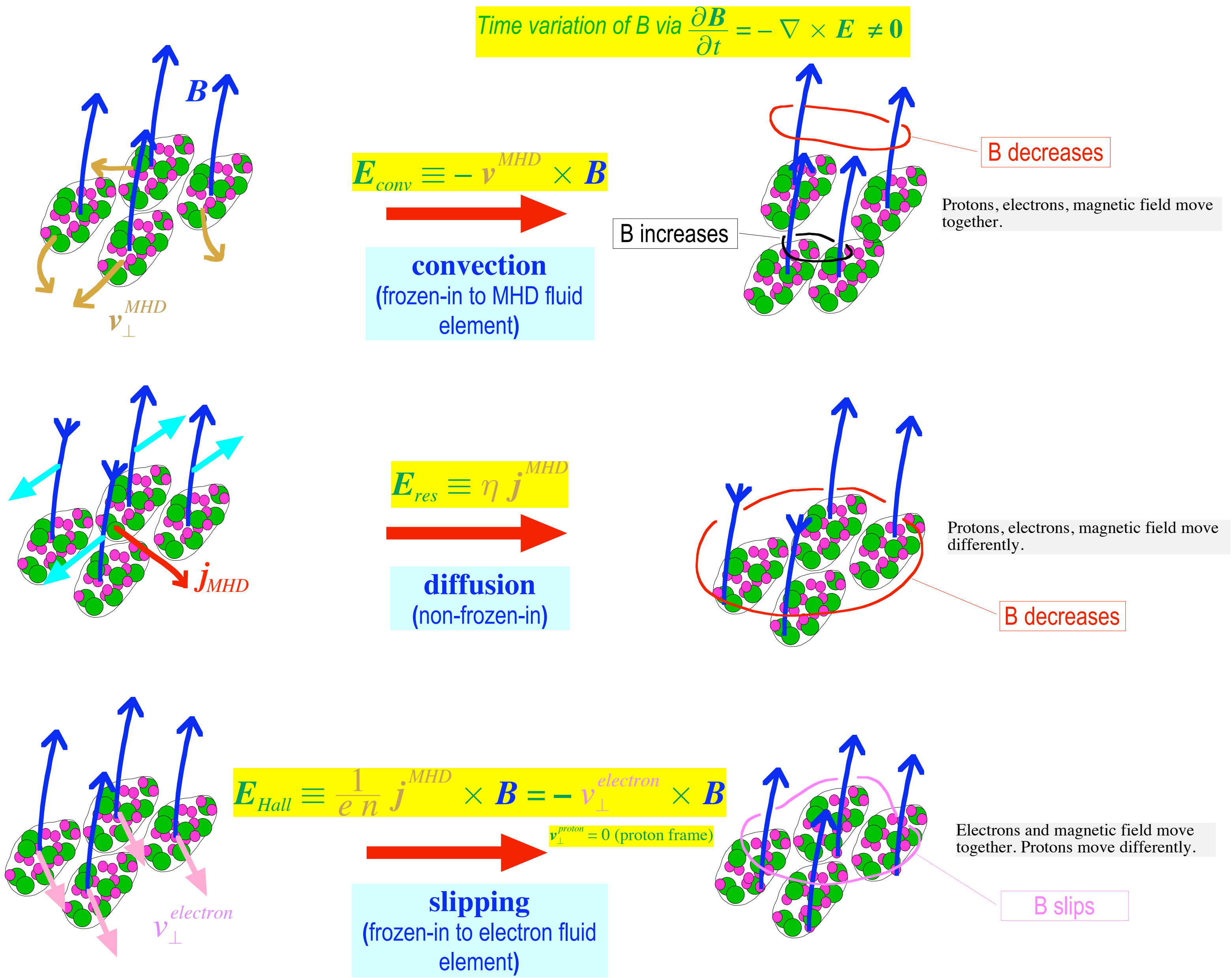
$$\boldsymbol{E}^{MHD} = -\,\boldsymbol{v}^{MHD}\times\boldsymbol{B} + \eta\,\boldsymbol{j}^{MHD} + \frac{1}{e\,n}\left[\boldsymbol{j}^{MHD}\times\boldsymbol{B} - \nabla P^e\right]$$

assumption

$\nabla P^e \sim 0$: Electron's inertia is **small enough** to **remove nonuniformity of electron's pressure during** t^{MHD}.

Ohm's law in MHD

(determines electric field from flow velocity & current density)



Time variation of B via $\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} \neq 0$
$\boldsymbol{B}$
$\boldsymbol{E}_{conv} \equiv -\boldsymbol{v}^{MHD} \times \boldsymbol{B}$
convection
(frozen-in to MHD fluid element)
$v_{\perp}^{MHD}$
B decreases
B increases
Protons, electrons, magnetic field move together.
$\boldsymbol{E}_{res} \equiv \eta\, \boldsymbol{j}^{MHD}$
diffusion
(non-frozen-in)
j_{MHD}
Protons, electrons, magnetic field move differently.
B decreases
$\boldsymbol{E}_{Hall} \equiv \frac{1}{e\,n}\, \boldsymbol{j}^{MHD} \times \boldsymbol{B} = -\, v_{\perp}^{electron} \times \boldsymbol{B}$
$\boldsymbol{v}_{\perp}^{proton} = 0$ (proton frame)
slipping
(frozen-in to electron fluid element)
$v_{\perp}^{electron}$
Electrons and magnetic field move together. Protons move differently.
B slips