Approximations used in MHD

• $v_0 << c$ (non-relativistic approximation)

Comparison between electromagnetic terms in the generalized Ampere's law

$$j = \frac{1}{\mu_0}\left(\nabla \times B - \frac{1}{c^2}\frac{\partial E}{\partial t}\right)$$

Faraday's law

$$\nabla \times E = -\frac{\partial B}{\partial t} \rightarrow \frac{E_0}{l_0} \sim \frac{B_0}{t_0}$$

$$\frac{B_0}{l_0} \qquad \frac{1}{c^2}\frac{E_0}{t_0} \longrightarrow \frac{1}{c^2}\frac{E_0}{t_0} \approx \frac{1}{c^2}\frac{1}{t_0}\frac{l_0}{t_0}B_0 = \frac{1}{c^2}\frac{l_0^2}{t_0^2}\frac{B_0}{l_0} = \frac{v_0^2}{c^2}\frac{B_0}{l_0} << \frac{B_0}{l_0}$$

$$\therefore \frac{B_0}{l_0} >> \frac{1}{c^2}\frac{E_0}{t_0} \Rightarrow we\ neglect\ \frac{1}{c^2}\frac{\partial E}{\partial t}$$

=> Propagation of electromagnetic waves (radiation) is not described by MHD.

$$\nabla \times B = \mu_0 j + \frac{1}{c^2}\frac{\partial E}{\partial t} \Rightarrow \nabla \times B = \mu_0 j$$

Ampere's law in MHD

(determine magnetic field from current density)

• $\rho_c \sim 0$ (local charge neutrality)

$$\nabla \cdot \boldsymbol{E} = \frac{\rho_c}{\varepsilon_0} \Rightarrow \frac{E_0}{l_0} \sim \frac{e\,(n_+ - n_-)}{\varepsilon_0}$$

$$n_+ - n_- \sim \frac{\varepsilon_0 E_0}{e\, l_0} \approx \frac{\varepsilon_0 \left(\dfrac{l_0 B}{t_0}\right)}{e\, l_0} = \frac{\varepsilon_0 v_0 B}{e\, l_0}$$

Condition of local charge neutrality:

$$n_+ - n_- << n_+ + n_- \equiv n_{total} \ (total\ number\ density)$$

$$\therefore \frac{\varepsilon_0 v_0 B}{e\, l_0} << n_{total} \quad \Rightarrow \quad 6.7\times10^{7} \frac{v_0 B}{l_0} \ll n_{total}$$

$e = 1.6\times10^{-19}$ C (electron charge)

e.g. *solar corona*: $n_{total} \sim 10^{14}$ m^{-3}, $v_0 \sim 10^{5}$ m/s, $l_0 \sim 10^{7}$ m, $B \sim 10^{-2}$ T

$$6.7\times10^{7} \frac{v_0 B}{l_0} << n_{total} \rightarrow 6.7\times10^{3} << 10^{14}$$ ***... satisfied***

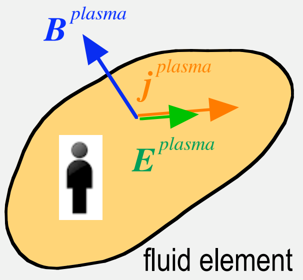
Coulomb's law in MHD $\nabla \cdot \boldsymbol{E} = \dfrac{\rho_c}{\varepsilon_0} \sim 0$

(does NOT determine electric field from charge density => **Ohm's law**)

Ohm's law

※ rough explanation => more detailed explanation is given in fluid part

Plasma frame



$$E^{plasma} = \eta\, j^{plasma} = \frac{1}{\sigma} j^{plasma}$$

η*... resistivity* $\left(\mathrm{ohm\ m} = \mathrm{kg\ m^{3}\ s^{-3}\ A^{-2}}\right)$

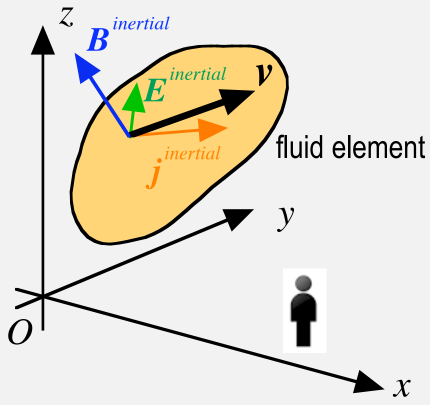
σ*... electric conductivity* $\left(\mathrm{ohm^{-1}\ m^{-1}} = \mathrm{mho\ m^{-1}}\right)$

=> **plasma quantities** determined by ***kinetic approach***

Ohm's law in plasma frame

Electric field is parallel to electric current density.

Inertial frame



When the fluid element moves at **constant velocity v** in inertial frame, then

$j^{inertial} = j^{plasma}$ *non-relativistic, charge-neutral case*

$E^{inertial} = E^{plasma} - v \times B^{plasma}$ $\quad B^{inertial} = B^{plasma}$

So,

$$E^{inertial} = \eta\, j^{inertial} - v \times B^{inertial}$$

Ohm's law in inertial frame

(determines electric field from flow velocity & current density)

$E_{convective} \equiv - v \times B^{inertial}$ ... transport magnetic field at $v_{\perp}$