Summary of motions of a gyrating particle

Standard form: $m\dfrac{d\boldsymbol{v}}{dt} = q\,\boldsymbol{v}_\perp \times \boldsymbol{B}_0 + \boldsymbol{F}_0$

$q\,\boldsymbol{v}_\perp \times \boldsymbol{B}_0$ ⟶ **gyration**

$\boldsymbol{F}_0$ **(external force)** ⟶ **drift, mirror effect**

$q\,\boldsymbol{v}_\perp \times \boldsymbol{B}$ ($|\boldsymbol{B}|$ **changes in** $\perp$*-direction*) ⟶ $q\,\boldsymbol{v}_\perp \times \boldsymbol{B}_0 + \boldsymbol{F}_0$

$$\boldsymbol{F}_0 = -\frac{m\,v_G^2}{2\,B_0}\nabla_\perp B$$

$q\,\boldsymbol{v}_\perp \times \boldsymbol{B}$ ($|\boldsymbol{B}|$ **changes in** *//-direction*) ⟶ $q\,\boldsymbol{v}_\perp \times \boldsymbol{B}_0 + \boldsymbol{F}_0$

$$\boldsymbol{F}_0 = -\frac{m\,v_G^2}{2\,B_0}\nabla_{//} B$$

Fundamental form: $\boldsymbol{F}_0 = -\nabla\left(-\boldsymbol{\mu}\bullet\boldsymbol{B}\right)$ $|\mu| \sim const.$

$$\boldsymbol{\mu} = -\frac{m\,v_G^2/2}{B_0}\,\boldsymbol{b}\,,\quad \boldsymbol{b} \equiv \frac{\boldsymbol{B}_0}{|\boldsymbol{B}_0|}\ \text{(unit vector of } \boldsymbol{B}_0\text{)}$$

diamagnetic

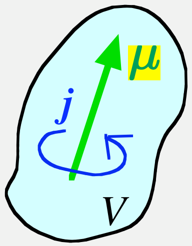
Magnetic moment of a charge particle

Magnetic (dipole) moment...

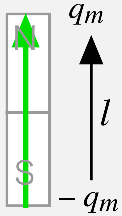
Definition of ***magnetic moment***:

$$\boldsymbol{\mu} \equiv \frac{1}{2}\int_V \boldsymbol{r} \times \boldsymbol{j}\, dr^3$$

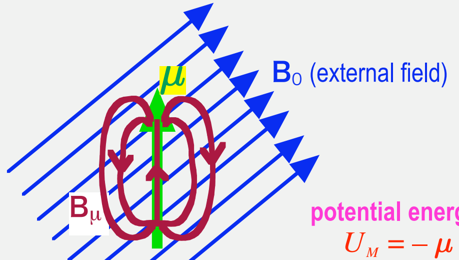
current-based definition



$$\boldsymbol{\mu} \equiv q_m \boldsymbol{l}$$



magnetic charge-based definition



$$\boldsymbol{B}_\mu = \frac{\mu_0}{4\pi}\left[\frac{3\,\boldsymbol{r}\,(\boldsymbol{\mu}\bullet\boldsymbol{r}) - r^2\boldsymbol{\mu}}{r^5} + \frac{8\pi}{3}\boldsymbol{\mu}\,\delta^3(\boldsymbol{r})\right]$$

potential energy: $U_M = -\boldsymbol{\mu}\cdot\boldsymbol{B}_0(\boldsymbol{r})$

force: $\boldsymbol{F} = -\nabla U_M$

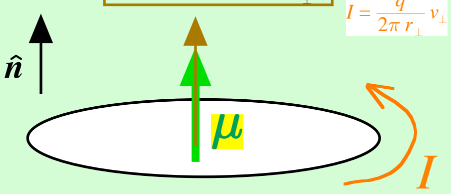
torque: $\boldsymbol{N} = \boldsymbol{\mu}\times\boldsymbol{B}_0$

Magnetic moments caused by ***rotating motions of charged particle***:

Circular current (conduction current)

$$B_\mu^{center} = \frac{\mu_0}{2}\frac{I}{r_\perp}\boldsymbol{n} = \frac{\mu_0}{2\pi}\frac{\mu}{r_\perp^3}\boldsymbol{\mu}$$

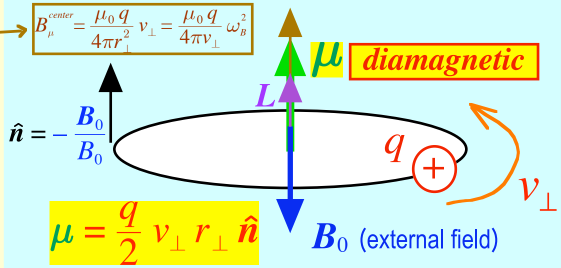
$$I = \frac{q}{2\pi r_\perp}v_\perp$$



$$\boldsymbol{\mu} = I\,S\,\hat{\boldsymbol{n}} = I\,\pi\,r_\perp^2\,\hat{\boldsymbol{n}}$$

Gyrating charged particle

$$B_\mu^{center} = \frac{\mu_0 q}{4\pi r_\perp^2}v_\perp = \frac{\mu_0 q}{4\pi v_\perp}\omega_B^2$$



$$\boldsymbol{\mu} = \frac{q}{2}v_\perp r_\perp\hat{\boldsymbol{n}}$$

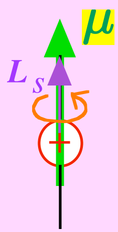
$$= \frac{q}{2m}\boldsymbol{L} \qquad v_\perp r_\perp \hat{\boldsymbol{n}} = \boldsymbol{L}/m$$

$$= I\,\pi\,r_\perp^2\,\hat{\boldsymbol{n}} \qquad I = \frac{q}{2\pi r_\perp}v_\perp$$

$$= -\frac{m v_\perp^2/2}{B_0}\frac{\boldsymbol{B}_0}{B_0} \qquad r_\perp = \frac{v_\perp}{\omega_B}$$

$$= -\frac{1}{2\pi}\frac{q^2}{m}\Phi_B\frac{\boldsymbol{B}_0}{B_0} \qquad \Phi_B = \pi r_\perp^2 B_0$$

Spinning charged particle



$$\boldsymbol{\mu} = g\,\frac{q}{2\,m}\boldsymbol{L}_S$$

spin angular momentum $\boldsymbol{L}_S$
... quantized by $\hbar$ or $\hbar/2$

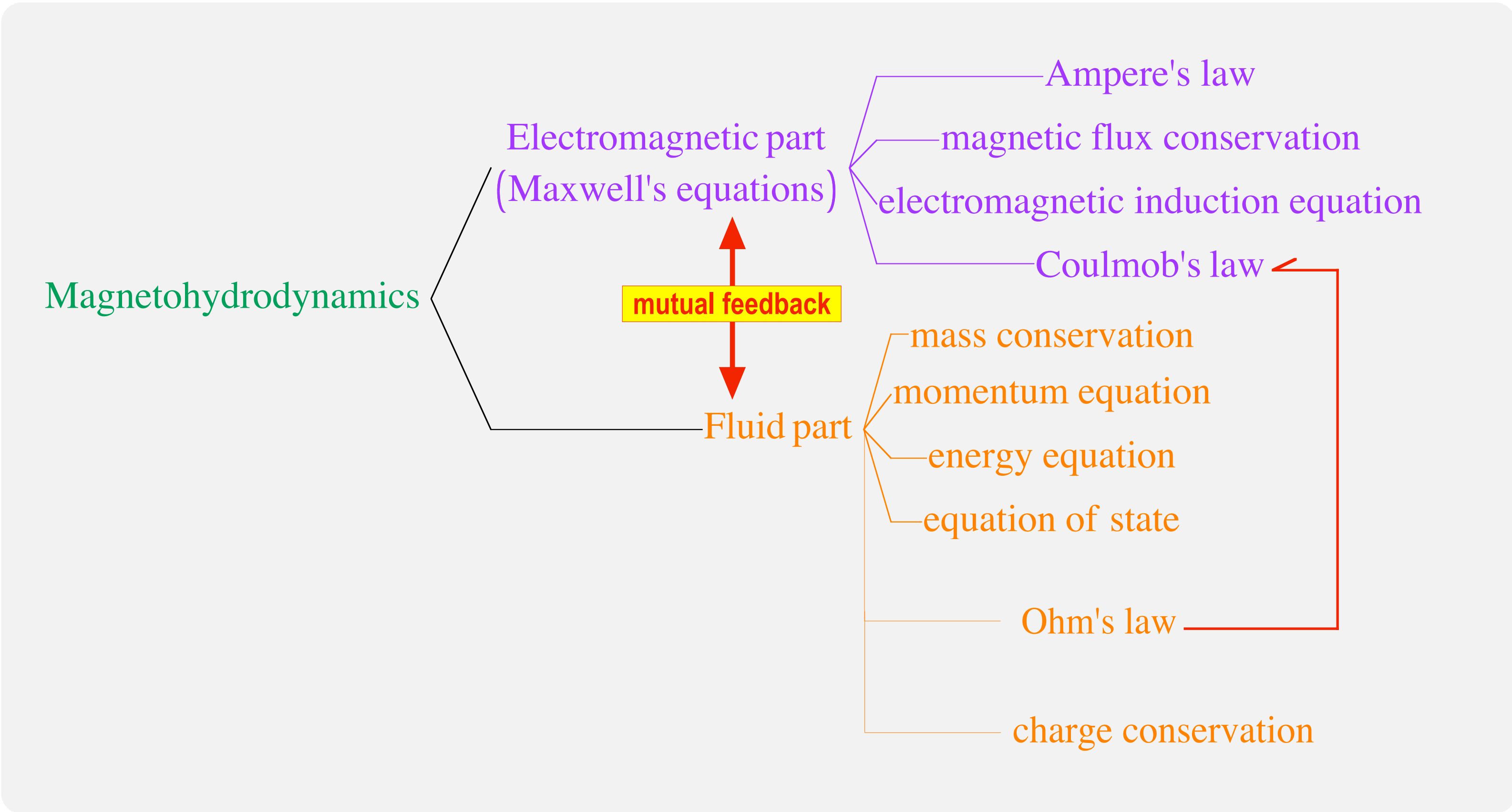
g... g-factor

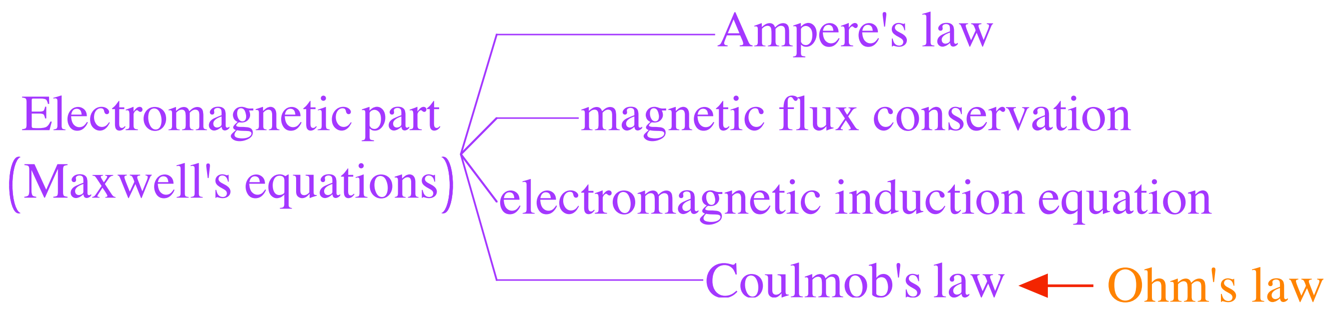
Magnetohydrodynamics

A fluid theory of plasmas

MHD equations...

Plasma (fluid) affects **electromagnetic fields**, while **electromagnetic fields** affect **plasma**.





(**Plasma** => **Fields**)

Maxwell's equations (determine electromagnetic fields from plasma quantities)

MKS unit

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$ **... Generalized Ampere's law**

$$\nabla \bullet \boldsymbol{B} = 0$$ **... Conservation of magnetic flux**

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$ **... Electromagnetic induction equation (Faraday's law)**

$$\nabla \bullet \boldsymbol{E} = \frac{\rho_c}{\varepsilon_0}$$ **... Coulomb's law**

$$\varepsilon_0 = 8.854\times10^{-12}\ \mathrm{F\ m^{-1}}$$ **... permittivity of free space**

$$\mu_0 = 4\pi\times10^{-7}\ \mathrm{H\ m^{-1}}$$ **... magnetic permeability**

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.998\times10^{8}\ \mathrm{m\ s^{-1}}$$ **... speed of light**

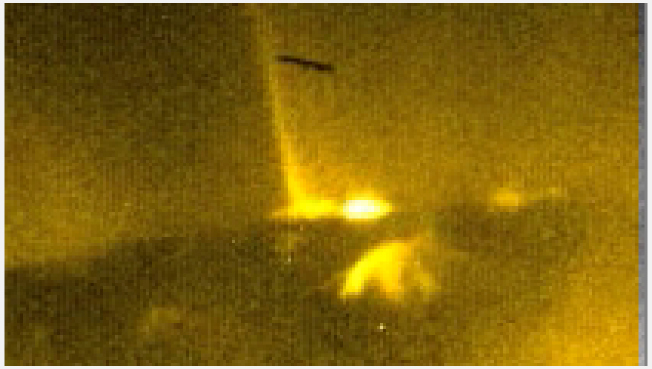
Unit:

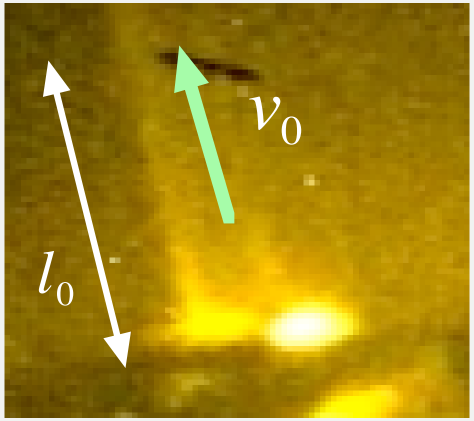
E... V/m, B... T (= 10^4 G), j... A/m^2

Typical scales used in MHD

MHD phenomena… ***large-scale***, ***slowly evolving*** *phenomena*

e.g. solar coronal jet





Typical scale:

length… $l_0 \sim 10^4$ km

velocity… $v_0 \sim 100$ km/s

time… $t_0 = \dfrac{l_0}{v_0} \sim 100$ s

r_G (gyration radius of a proton) ~ 1 m

τ_G (gyration period of a proton) ~ 10^{-4} s

in the solar corona