$$F_z \approx - \frac{m v_\theta^2}{2 B(0,z)} \frac{\partial B(0,z)}{\partial z}$$

general form

$$\boldsymbol{F}_{//} \approx -\mu \nabla_{//} B$$

$$|\mu| = \frac{\frac{1}{2} m v_\perp^2}{B} \text{ (magnitude of magnetic moment)}$$

$(v_\perp^2 = v_r^2 + v_\theta^2 \approx v_\theta^2$ is assumed$)$

※ Check the slide "**Magnetic moment**"

$$\boldsymbol{F} = -\nabla U_M = -\nabla(-\boldsymbol{\mu} \bullet \boldsymbol{B}) = -|\mu| \nabla B$$

$$\boldsymbol{\mu} = -|\mu| \frac{\boldsymbol{B}}{B} \quad |\mu| \sim const.$$

gyrating charged particle
=> diamagnetic magnetic moment

Equation of motion in $B_{//}$– *direction*: $m \dfrac{d \boldsymbol{v}_{//}}{dt} = \boldsymbol{F}_{//} \approx -\mu \nabla_{//} B$

take an inner product with $\boldsymbol{v}_{//}$

$$\boldsymbol{v}_{//} \bullet \nabla_{//} B = \frac{dB}{dt} - \underbrace{\frac{\partial B}{\partial t}}_{=0} = \frac{dB}{dt}$$

Static field (*B does not change with time*)

$(|\boldsymbol{v}_\perp \bullet \nabla_\perp B| \ll |\boldsymbol{v}_{//} \bullet \nabla_{//} B|$ is assumed$)$

$$\frac{d}{dt}\left(\frac{1}{2} m v_{//}^2\right) \sim -\mu \frac{dB}{dt}$$

$$\frac{d}{dt}(\mu B) = \frac{d}{dt}\left(\frac{m v_\perp^2}{2B} B\right) = \frac{d}{dt}\left(\frac{m v_\perp^2}{2}\right) = -\frac{d}{dt}\left(\frac{1}{2} m v_{//}^2\right)$$

Conservation of total kinetic energy: $\dfrac{d}{dt}\left(\dfrac{1}{2} m v_{//}^2 + \dfrac{1}{2} m v_\perp^2\right) = 0$

Lorentz force does not do any work on a particle.

$$v_{//}^2 + v_\perp^2 = v^2 = const.$$

$$\frac{d\mu}{dt} \sim 0$$

Conservation of magnetic moment

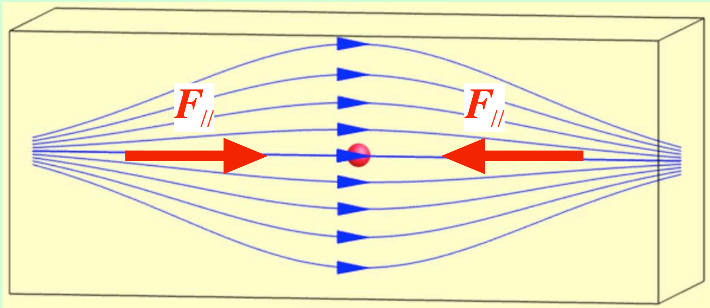
When the spatial variation of **B** felt by a charged particle while taking one gyration is sufficiently small,

its **magnetic moment** μ becomes an ***adiabatic invariant***: $\dfrac{d\mu}{dt} \sim 0$ → $\mu = \dfrac{m v_{\perp}^2/2}{B} = const.$

μ becomes particle's unchanged property such as m and q (although it depends on $\boldsymbol{B}$).

$$\frac{d}{dt}\left(\frac{1}{2} m v_{/\!/}^2\right) \sim -\mu \frac{dB}{dt}$$ → when B increases, $v_{/\!/}$ decreases.

μ and m are constant



$v_{/\!/}^2 + v_{\perp}^2 = v^2 = const.$

←	→
B increases $v_{/\!/}$ decreases $v_{\perp}$ increases	B increases $v_{/\!/}$ decreases $v_{\perp}$ increases

When $v_{/\!/}$ becomes 0, $\boldsymbol{F}_{/\!/} \approx -\mu \nabla_{/\!/} B$ causes reflection of the particle.

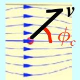
Mirror effect

ϕ_c: ***critical angle***

$$\frac{m\,(v \sin \phi_c)^2}{2\,B_{min}} = \frac{m\left(v \sin\left(\frac{\pi}{2}\right)\right)^2}{2\,B_{max}} = const.$$

central point with $B = B_{min}$ & $\phi = \phi_c$

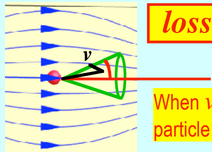
$v_{/\!/} = 0$ (*reflection point*) with $B = B_{max}$ & $\phi = \frac{\pi}{2}$



$$\sin^2 \phi_c = \frac{B_{min}}{B_{max}}$$

$\phi > \phi_c \Rightarrow$ *reflect*

$\phi < \phi_c \Rightarrow$ *escape*



loss cone $(\phi < \phi_c)$

When $v_{/\!/}$ is relatively large, the particle may enter a loss cone.

Example: **Magnetosphere**