

# Single particle motion in a given field

*( $B_{||}$ -direction)*

# Mirror effect

$\mathbf{B}$ : magnetic field (**nonuniform** (weak variation) & constant, almost straight shape)

$\mathbf{v}$ : particle's velocity  
 $m, q$ : particle's mass & charge

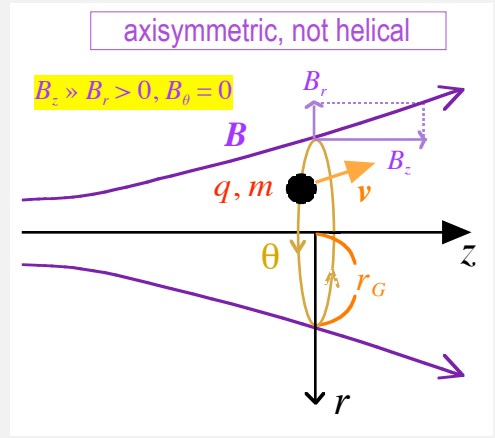
## Gyration-average Lorentz force $\langle q \mathbf{v} \times \mathbf{B} \rangle$ :

MKS unit

$$\begin{aligned} \mathbf{F} &= q (\mathbf{v}_r \hat{\mathbf{r}} + \mathbf{v}_\theta \hat{\boldsymbol{\theta}} + \mathbf{v}_z \hat{\mathbf{z}}) \times (\mathbf{B}_r \hat{\mathbf{r}} + \mathbf{B}_z \hat{\mathbf{z}}) \\ &= q (\mathbf{v}_r \mathbf{B}_z \hat{\mathbf{r}} \times \hat{\mathbf{z}} + \mathbf{v}_\theta \mathbf{B}_r \hat{\boldsymbol{\theta}} \times \hat{\mathbf{r}} + \mathbf{v}_\theta \mathbf{B}_z \hat{\boldsymbol{\theta}} \times \hat{\mathbf{z}} + \mathbf{v}_z \mathbf{B}_r \hat{\mathbf{z}} \times \hat{\mathbf{r}}) \\ &= q [\mathbf{v}_\theta \mathbf{B}_z \hat{\mathbf{r}} + (\mathbf{v}_z \mathbf{B}_r - \mathbf{v}_r \mathbf{B}_z) \hat{\boldsymbol{\theta}} - \mathbf{v}_\theta \mathbf{B}_r \hat{\mathbf{z}}] \end{aligned}$$

$\langle \mathbf{F} \rangle_{\text{gyration}} = |q| |\mathbf{v}_\theta| B_r \hat{\mathbf{z}}$   
 $F_z \approx |q| |\mathbf{v}_\theta| B_r(r_G, z)$

$\langle \mathbf{r} \rangle = 0, \langle \boldsymbol{\theta} \rangle = 0$      $q > 0 \rightarrow \mathbf{v}_\theta < 0$



## Taylor expansion of $B_r(r, z)$ around $r = 0$ :

odd function of  $r$

$$B_r(r, z) = \underbrace{B_r(0, z)}_{=0} + \left[ \frac{\partial B_r(r, z)}{\partial r} \right]_{r=0} r + \dots \approx 0$$

1st-order term is considered within  $0 \leq r \leq r_G$

$$\left[ \frac{\partial B_r}{\partial r} \right]_{r=0} \approx \left[ \frac{\partial B_r}{\partial r} \right]_{r=r_G}$$

$$\frac{B_r(r_G, z)}{r_G} \approx \left[ \frac{\partial B_r(r, z)}{\partial r} \right]_{r=0}$$

## Taylor expansion of $B_z(r, z)$ around $r = 0$ :

even function of  $r$

$$B_z(r, z) = B_z(0, z) + \left[ \frac{\partial B_z(r, z)}{\partial r} \right]_{r=0} r + \dots = B(0, z)$$

0th-order term is considered within  $0 \leq r \leq r_G$

1st-order term is 0 when  $r \rightarrow 0$  (smoothness)

## Divergence free condition ( $\nabla \cdot \mathbf{B} = 0$ ) at $r = r_G$ :

$$\nabla \cdot \mathbf{B} \Big|_{r=r_G} = \left[ \frac{\partial B_r(r, z)}{\partial r} \right]_{r=r_G} + \frac{B_r(r_G, z)}{r_G} + \left[ \frac{\partial B_z(r, z)}{\partial z} \right]_{r=r_G} = 0$$

$$\frac{B_r(r_G, z)}{r_G} \approx -\frac{1}{2} \left[ \frac{\partial B_z(r, z)}{\partial z} \right]_{r=r_G} \approx -\frac{1}{2} \frac{\partial B(0, z)}{\partial z}$$

$$\nabla \cdot \diamond = \frac{\partial}{\partial r} (\diamond_r) + \frac{\diamond_r}{r} + \frac{1}{r} \frac{\partial \diamond_\theta}{\partial \theta} + \frac{\partial \diamond_z}{\partial z}$$

Formula in cylindrical coordinates

$$B_r(r_G, z) \approx -\frac{r_G}{2} \frac{\partial B(0, z)}{\partial z} \rightarrow F_z \approx -\frac{m v_\theta^2}{2 B(0, z)} \frac{\partial B(0, z)}{\partial z}$$

$$r_G = \frac{|\mathbf{v}_\theta|}{\omega_B}, \quad \omega_B = \frac{|q| B(0, z)}{m}$$