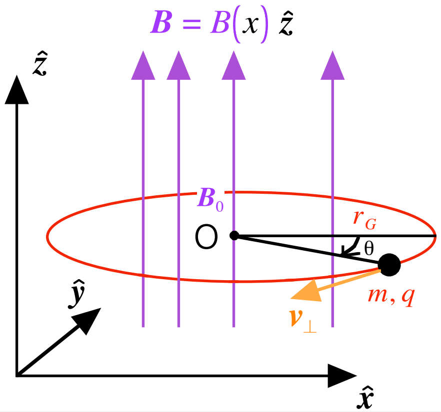
Gradient-B drift



$\boldsymbol{B}$: magnetic field (**nonuniform** (weak variation) & constant, straight shape)

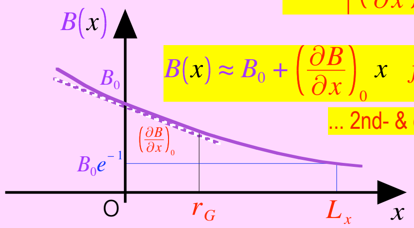
$\boldsymbol{v}$: particle's velocity m, q: particle's mass & charge

$\boldsymbol{B} = B(x)\ \hat{z}$ => weak variation: $L_x \equiv \dfrac{B_0}{\left|\left(\dfrac{\partial B}{\partial x}\right)_0\right|} \gg r_G$

$\dfrac{\left|\left(\frac{\partial B}{\partial x}\right)_0 r_G\right|}{B_0} \ll 1 \Rightarrow r_G \ll L_x$

$B(x) \approx B_0 + \left(\dfrac{\partial B}{\partial x}\right)_0 x \quad for\ |x| \le r_G \ll L_x$

... 2nd- & even higher-order terms are neglected.



MKS unit

Equation of motion in $B_\perp$–plane: $m \dfrac{d\boldsymbol{v}_\perp}{dt} = q\ \boldsymbol{v}_\perp \times \boldsymbol{B}$

$\langle \cos^2\theta \rangle = \dfrac{\int_0^{2\pi} \cos^2\theta\, d\theta}{\int_0^{2\pi} d\theta} = \dfrac{1}{2}$

Gyration-average (remove gyration effect): $\langle f(\theta) \rangle = \dfrac{\int_0^{2\pi} f(\theta)\, d\theta}{\int_0^{2\pi} d\theta}$

=> only x-component of force is taken into account

$r_G = \dfrac{v_G}{\omega_B} = \dfrac{v_G\, m}{q\, B_0}$ $\left(\dfrac{\partial B}{\partial x}\right)_0 \hat{x} = \nabla_0 B$

$\boldsymbol{B}(x) \approx \boldsymbol{B}_0 + \left(\dfrac{\partial B}{\partial x}\right)_0 x\ \hat{z}$

$q\ \boldsymbol{v}_\perp \times \boldsymbol{B} \rightarrow q\ \boldsymbol{v}_\perp \times \boldsymbol{B}_0 + q\ \boldsymbol{v}_\perp \times \left[\left(\dfrac{\partial B}{\partial x}\right)_0 x\ \hat{z}\right]$

$\boldsymbol{F}_{\nabla B} = -\dfrac{m\, v_G^2}{2\, B_0} \nabla_0 B$: **gradient-B force**

$\boldsymbol{F}_{\nabla B} = -\mu\ \nabla_0 B$ $\mu = \dfrac{m\, v_G^2 / 2}{B_0}$

$\left(\dfrac{\partial B}{\partial x}\right)_0 r_G \cos\theta\ \hat{z}$

$\boldsymbol{v}_\perp = \boldsymbol{v}_G(t) + \Delta\boldsymbol{v}$ → $\boldsymbol{v}_\perp \approx -v_G \sin\theta\ \hat{x} - v_G \cos\theta\ \hat{y}$

assumption: $|\boldsymbol{v}_G(t)| \gg |\Delta\boldsymbol{v}|$

justification $\dfrac{v_{\nabla B}}{v_G} \sim \dfrac{v_G}{\omega_B} \dfrac{1}{L_x} = \dfrac{r_G}{L_x} \ll 1$

when $\boldsymbol{F}_{\nabla B}$ is uniform & constant

=> **gradient-B drift**

$\boldsymbol{v}_{\nabla B} = \dfrac{\boldsymbol{F}_{\nabla B} \times \boldsymbol{B}_0}{q\, B_0^2}$

$= -\dfrac{m\, v_G^2}{2\, q\, B_0^3} (\nabla_0 B) \times \boldsymbol{B}_0$

$\boldsymbol{v}_\perp = \boldsymbol{v}_G(t)$ **(gyration)** + $\boldsymbol{v}_{\nabla B}$ **(grad-B drift)**