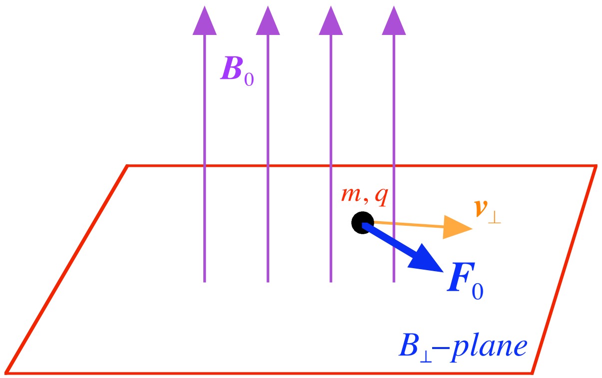
Single particle motion in a given field

($B_\perp$-*plane*)

Drift of a gyrating particle by external force



B_0: magnetic field (uniform & constant, straight shape)

v: particle's velocity m, q: particle's mass & charge

F_0: external force (uniform & constant, in $B_\perp$–plane)

MKS unit

Equation of motion in $B_\perp$–plane: $m \dfrac{d\boldsymbol{v}_\perp}{dt} = q\, \boldsymbol{v}_\perp \times \boldsymbol{B}_0 + \boldsymbol{F}_0$

$\boldsymbol{v}_\perp = \boldsymbol{v}_G(t) + \boldsymbol{v}_F$

$m \dfrac{d\boldsymbol{v}_\perp}{dt} = q\, \boldsymbol{v}_\perp \times \boldsymbol{B}_0 + \boldsymbol{F}_0$ →

$$\begin{cases} \dfrac{d^2 \boldsymbol{v}_G(t)}{dt^2} = -\dfrac{q^2 B_0^2}{m^2} \boldsymbol{v}_G(t) \\ \boldsymbol{v}_F = \dfrac{\boldsymbol{F}_0 \times \boldsymbol{B}_0}{q B_0^2} \end{cases}$$

$q\, \boldsymbol{v}_G(t) \times \boldsymbol{B}_0$ => gyration: $\boldsymbol{v}_G(t)$

gyro-frequency: $\omega_B \equiv \dfrac{q B_0}{m}$

gyration radius: $r_G = \dfrac{v_\perp}{\omega_B}$

$\boldsymbol{F}_0$ => drift: $\boldsymbol{v}_F$

e.g.) $\boldsymbol{F}_0 = q\, \boldsymbol{E}_0$

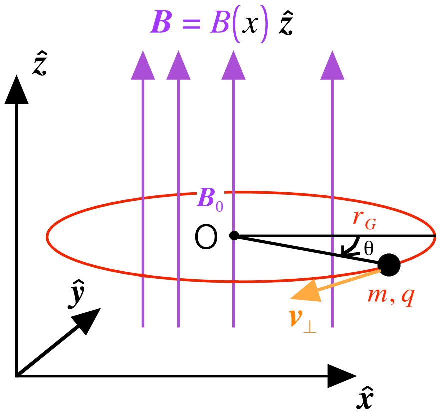
=> ExB drift ($\boldsymbol{v}_{E\times B} = \dfrac{\boldsymbol{E}_0 \times \boldsymbol{B}_0}{B_0^2}$)

$\boldsymbol{v}_\perp = \boldsymbol{v}_G(t)$ **(gyration)** + $\boldsymbol{v}_F$ **(drift)**

Gradient-B drift

$\boldsymbol{B}$: magnetic field (**nonuniform** (**weak variation**) & constant, straight shape)

$\boldsymbol{v}$: particle's velocity $\quad m, q$: particle's mass & charge



$\boldsymbol{B} = B(x)\,\hat{z}$ => weak variation: $L_x \equiv \dfrac{B_0}{\left|\left(\dfrac{\partial B}{\partial x}\right)_0\right|} \gg r_G$

$$\frac{\left|\left(\frac{\partial B}{\partial x}\right)_0 r_G\right|}{B_0} \ll 1 \;\Rightarrow\; r_G \ll L_x$$

$$B(x) \approx B_0 + \left(\frac{\partial B}{\partial x}\right)_0 x \quad for\ |x| \le r_G \ll L_x$$

... 2nd- & even higher-order terms are neglected.