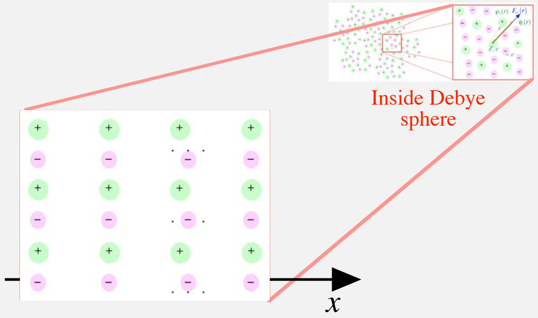
Plasma frequency

1 dimensional model



$\frac{d}{dt} \Leftrightarrow \frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla$

Subscript 0 indicates initial equilibrium values, while subscript 1 indicates oscillating values.

Ions are static ($n_i = n_0,\ v_{ix} = 0$) because their mass is large.

Electrons oscillate. Electric field also oscillates.

$n_e = n_0 + n_1(x,t)$ $v_{ex} = v_0 + v_1(x,t),\ v_0 = 0$ $E_x = E_0 + E_1(x,t),\ E_0 = 0$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}\left(n_e v_{ex}\right) = 0$$

$$m_e n_e \left[\frac{\partial v_{ex}}{\partial t} + v_{ex}\frac{\partial v_{ex}}{\partial x}\right] = -e\, n_e E_x$$

$$\frac{\partial E_x}{\partial x} = 4\pi e\left(n_i - n_e\right)$$

Linearization

$\Diamond_1 \Delta_1 \sim 0$

$\Diamond_1 \propto e^{i(k_x x - \omega t)}$

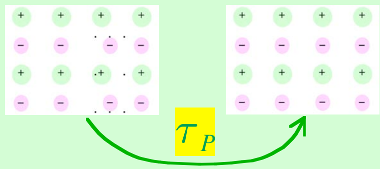
$\frac{\partial \Diamond_1}{\partial t} \Rightarrow -i\omega \Diamond_1,\ \frac{\partial \Diamond_1}{\partial x} \Rightarrow i k_x \Diamond_1$

Plasma frequency:

$$\nu_p = \frac{\omega_p}{2\pi}$$

$\omega = \omega_p \equiv \sqrt{\frac{4\pi n_0 e^2}{m_e}}$ $\omega_p = \sqrt{\frac{n_0 e^2}{\varepsilon_0 m_e}}$ (MKS unit)

$\tau_P = \frac{1}{\omega_P} = \frac{L_D}{v_{Te}}$ ... A time required for an electron to relax to a neutral state via oscillation => **Neutralization time**



Electron feels Coulombic electric field during τ_P (or the electric field exists for the electron during τ_P).

Characteristic scales related to magnetic field

(gyration)

In plasmas, charge density is always close to zero, while electric current density is not.

$$\rho_c = e\,(Z_i\, n_i - n_e) \sim 0$$

$$E_{coulomb}(\mathbf{r}) = \iiint_{V_0} \frac{\rho_C(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\, dV'$$

CGS unit

Coulombic electric field **does not exist globally & continuously.**

$$\mathbf{j} = Z_i\, n_i\, e\, \mathbf{v}_i + n_e\,(-e)\,\mathbf{v}_e \sim e\, n_e\,(\mathbf{v}_i - \mathbf{v}_e) \neq \mathbf{0}$$

$$\mathbf{B}(\mathbf{r}) = \frac{1}{c^2}\iiint_{V_0} \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\, dV'$$

CGS unit

Magnetic field **does exist globally & continuously.**

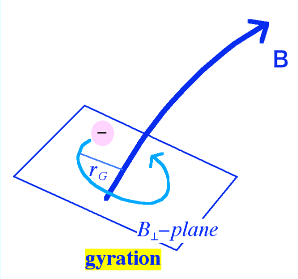
Cyclotron (gyro-) frequency… charged particle gyrates around a magnetic field line

$$\nu_B = \frac{\omega_B}{2\pi}$$

$$m\,\frac{d\mathbf{v}_\perp}{dt} = \frac{q}{c}\,\mathbf{v}_\perp \times \mathbf{B} \;\Rightarrow\; m\, r_G\, \omega_B^2 = \frac{q}{c}\, r_G\, \omega_B\, B \;\longrightarrow\; \omega_B \equiv \frac{q\,B}{m\,c}$$

$$v_\perp = r_G\, \omega_B$$

$$\omega_B \equiv \frac{q\,B}{m}\ \text{(MKS unit)}$$



Magnetic field also introduces characteristic length & time scales into plasma systems…

Length scale… **gyration radius:** $r_G = \dfrac{v_\perp}{\omega_B} = \dfrac{v_T}{\omega_B}$

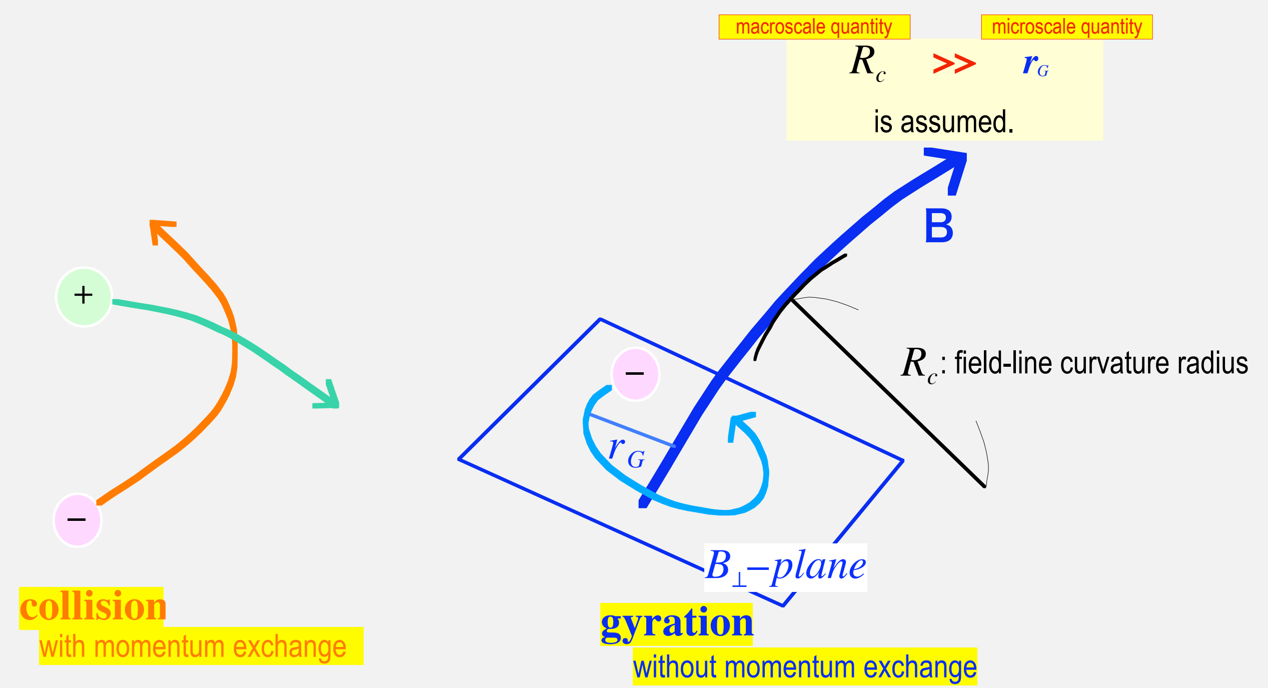
When $r_G << l_{mfp}$, magnetic field plays a main role in **changing the momentum** of a particle **in B⊥-plane**.

$v_\perp = v_T$ for thermal plasma

Time scale… **gyration time:** $\tau_G = \dfrac{1}{\omega_B} = \dfrac{r_G}{v_T}$

When $\tau_G << \tau_c$, a particle is **strongly coupled** with magnetic field.

Collision vs. Gyration



Gyration plays a collision-like role in changing the momentum of a charged particle in $B_\perp - plane$.

Difference between collision & gyration:

collision produces **random motion** in **all directions** => **pressure**, **temperature** in **3D space**
(random motion: direction & magnitude of v are changed; pressure, temperature: with thermalization)

gyration produces **ordered motion** in $B_\perp - plane$ => **pressure**, **current** in $B_\perp - plane$
(ordered motion: only direction of v is changed; pressure: without thermalization; current: when nonuniformity exists)