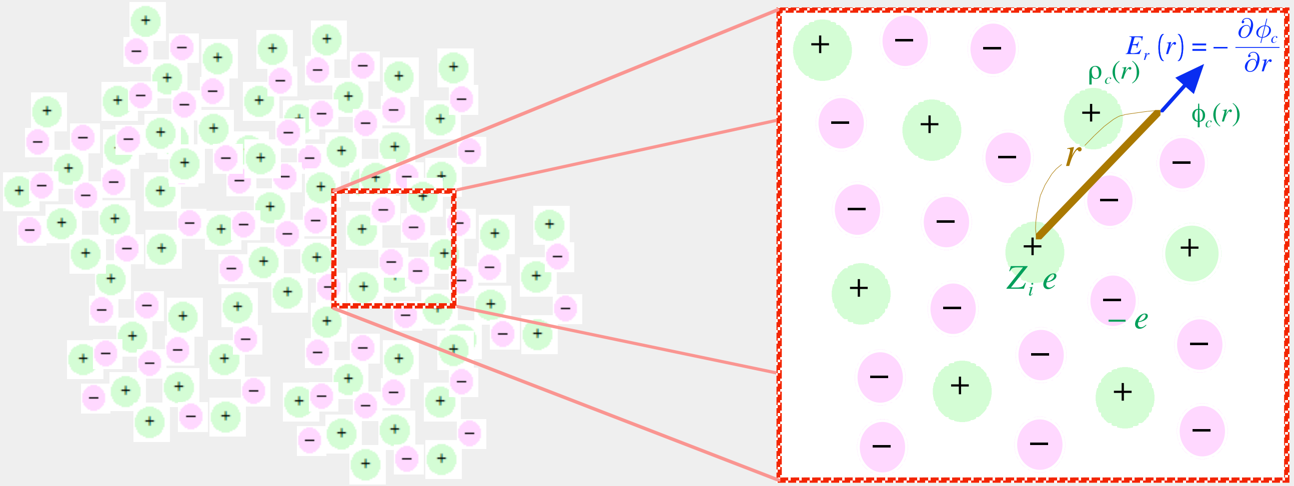
Debye length

(shielding of Coulombic electric field)

Debye length

Isotropic distribution model... only depends on the distance r from the central particle

(each particle continuously moves, while keeping the same isotropic distribution)



Z_i... *ion's valency number*
($Z_i = 1$ for a proton)

ϕ_c: *electrostatic potential*, ρ_c: *charge density*... both depend on r

Equation for electrostatic potential... $\nabla^2 \phi_c = -4\pi\rho_c(r)$ CGS unit

Thermal state is considered: $n(r) = n_e \dfrac{e^{-\frac{(-e)\phi_c}{k_B T_e}}}{e^{-\frac{0}{k_B T_e}}}$

Charge density distribution (thermal state) => $\rho_c(r) = Z_i\, e\, \delta\,(r) + (-e)\, n_e\, e^{-\frac{(-e)\phi_c}{k_B T_e}} + (Z_i\, e)\, n_i\, e^{-\frac{(Z_i e)\phi_c}{k_B T_i}}$

central ion | surrounding particles

n_e, n_i... *number density* (constant) *of electron and ion* when $\phi_c \to 0\ (r \to \infty)$

Assumption:
electrostatic energy « thermal energy
$|-e\,\phi_c| \ll k_B T_e,\ |Z_i\, e\,\phi_c| \ll k_B T_i$

$e^x \approx 1 + x$ *when* $|x| \ll 1$

$Z_i\, n_i = n_e$

$T \equiv \dfrac{T_e (Z_i T_i)}{T_e + (Z_i T_i)}$ reduced temperature

$\Rightarrow -\dfrac{1}{4\pi L_D^2}\phi_c$

Debye length

$L_D \equiv \sqrt{\dfrac{k_B T}{4\pi e^2 n_e}}$

MKS unit: $L_D \equiv \sqrt{\dfrac{\varepsilon_0 k_B T}{e^2 n_e}}$

Physical meaning of L_D => Debye shielding of Coulombic electric field

$$\nabla^2 \phi_c = -4\pi\rho_c(r) \longrightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi_c}{dr}\right) = -4\,\pi\,Z_i\,e\,\delta(\boldsymbol{r}) + \frac{\phi_c}{L_D^2}$$

spherical coordinate system

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

isotropic distribution: $\frac{\partial}{\partial\theta}=0,\ \frac{\partial}{\partial\phi}=0$

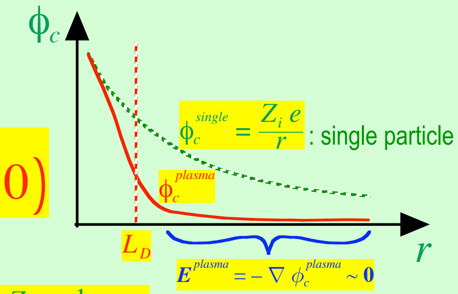
Transform the function: $\Phi = r\,\phi_c$ => $\frac{d^2\Phi}{dr^2} = \frac{\Phi}{L_D^2}\ (0<r<\infty)$

Boundary condition:

$$\phi_c \rightarrow \frac{Z_i\,e}{r}\ when\ r\rightarrow 0,\ \ \phi_c \rightarrow 0\ when\ r\rightarrow\infty$$

Solution:

$$\phi_c^{plasma} = \frac{Z_i\,e}{r}\,e^{-\frac{r}{L_D}}\ \ (r>0)$$



$\rho_c^{plasma}(r) \sim 0$

$$\rho_c^{plasma}(r) \sim Z_i\,e\,\delta(r) - \frac{1}{4\pi\,L_D^2}\,\phi_c^{plasma} = Z_i\,e\,\delta(r) - \frac{Z_i\,e}{4\pi\,L_D^2}\,\frac{1}{r}\,e^{-\frac{r}{L_D}}$$

=> when $r \gg L_D$, $\rho_c^{plasma}(r) \sim 0$ (**local charge neutrality**)

No global Coulombic electric field exists in **plasma systems.**

Surrounding ions and electrons **shield** Coulombic electric field generated by each ion and electron, which significantly reduces the effective range of Coulomb force between them in plasma systems.

$$L_D \equiv \sqrt{\frac{k_B\,T}{4\pi e^2\,n_e}} \quad T \equiv \frac{T_e\,(Z_i\,T_i)}{T_e + (Z_i\,T_i)}$$

→ Ion with a large valency number shields the electric field weakly (T & L_D increase).