Summary of plasma state, kinetic & fluid approaches

Plasma state... 4th state of matter in a system where

there are **many charged particles**

these particles **behave collectively** (=> local charge neutrality)

Kinetic approach... more fundamental approach in the sense that

it is based on **real object** (**particle**)

it can be used **even for non-plasma state** (each particle behaves independently)

Fluid approach... less fundamental approach in the sense that

it is based on **virtual object** (**fluid element**)

it can be used **only for thermal state** (velocity distribution of particles is given by Maxwellian distribution)

Characteristic scales of plasmas

MKS	CGS
B	$\frac{B}{c}$
ε_0	$\frac{1}{4\pi}$
μ_0	$\frac{4\pi}{c^2}$
e	$e\sqrt{4\pi\varepsilon_0}$

Characteristic scales related to electric field

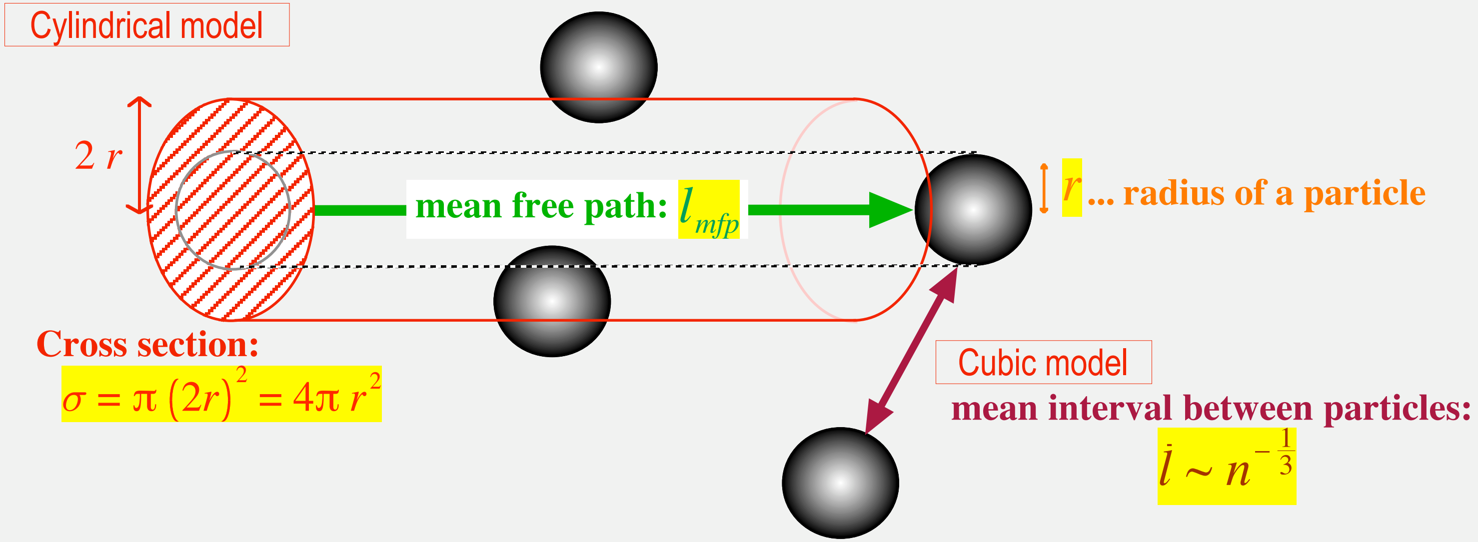
(collision, oscillation)

Mean free path

(collision)

Mean free path of a neutral particle (collision with a direct touch)

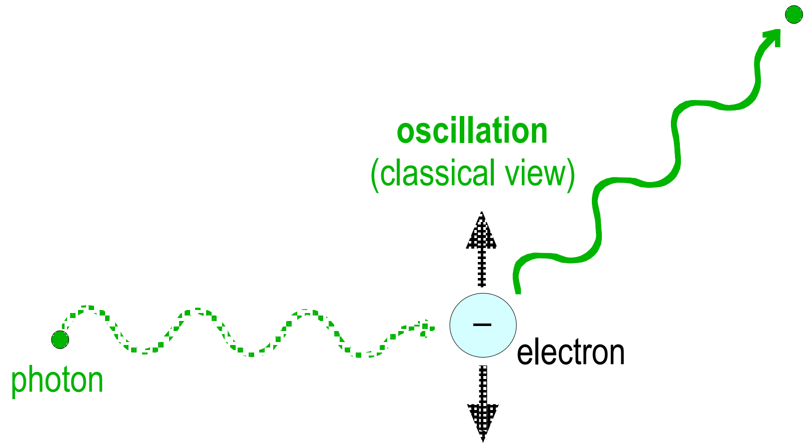
… A distance over which a particle freely travels without colliding with another particle



The number of particles inside the cylinder with a volume of $V = \sigma\, l_{mfp}$ is 1.

$$n\,\sigma\, l_{mfp} = 1 \longrightarrow l_{mfp} = \frac{1}{n\,\sigma}$$

Mean free path of a photon (collision between a photon and an electron)



Thomson formula for cross section:

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = \frac{8\pi}{3} r_e^2 = \frac{2}{3}\left[\pi\,(2\,r_e)^2\right]$$

$r_e \equiv \dfrac{e^2}{m_e c^2}$ ... classical radius of electron $\sim 2.8\times10^{-15}$ m

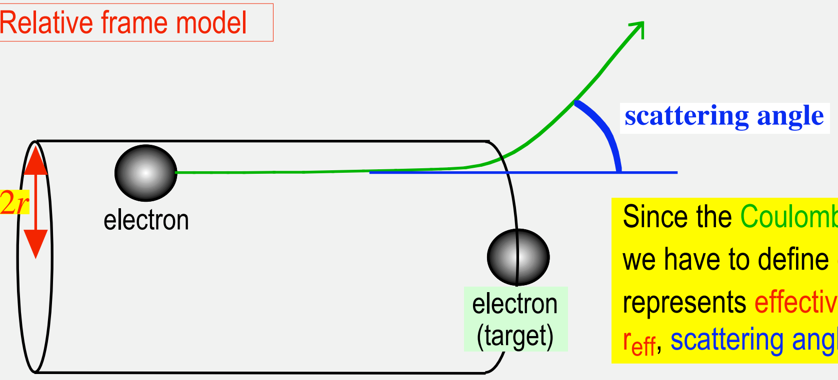
electron's rest energy ($m_e c^2$) ~ its Coulombic energy ($\frac{e^2}{r_e}$)

Photon's mean free path (***Thomson scattering***): $l_T = \dfrac{1}{n_e\,\sigma_T}$

Mean free path of a charged particle (collision without a direct touch)

General definition of collision: **A process causing a change in momentum** (or **velocity**)

Relative frame model



Since the Coulomb force between the electrons is long-range force, we have to define effective radius of Coulombic collision r_{eff} that represents effective collision with a large scattering angle (when $r \sim r_{eff}$, scattering angle ~ 90°).

How to define r_{eff} => **electron's kinetic energy ($m_e v_e^2$) ~ its Coulombic energy ($\frac{e^2}{r_{eff}}$)**

$$\Rightarrow r_{eff} \sim \frac{e^2}{m_e v_e^2}$$

$$r_{eff} \sim \frac{e^2}{m_e v_e^2} \xrightarrow[m_e v_e^2 \sim k_B T_e]{\text{thermal plasma}} r_{eff} \sim \frac{e^2}{k_B T_e} \sim 10^{-11}\ \mathrm{m} \quad when\ T_e = 10^6\ K$$

$$\gg r_e \sim 10^{-15}\ \mathrm{m}$$

$$\sigma^{e-e} \sim \pi (2r_{eff})^2 \sim 4\pi \left(\frac{e^2}{m_e v_e^2}\right)^2 \Rightarrow l_{mfp}^{e-e} \equiv \frac{1}{n_e \sigma^{e-e}} \sim \frac{m_e^2 v_e^4}{4\pi n_e e^4} \xrightarrow[m_e v_e^2 \sim k_B T_e]{\text{thermal plasma}} l_{mfp}^{e-e} \sim \frac{(k_B T_e)^2}{4\pi n_e e^4}$$

* More precisely (including non-effective collision), $l_{mfp}^{e-e} \sim \frac{m_e^2 v_e^4}{4\pi n_e e^4} \frac{1}{\ln \Lambda}$, where $\ln \Lambda \sim 10$ (**Coulomb logarithm**)