Kinetic approach

(in the case of mechanics)

$$\begin{cases} m\dfrac{dv_x}{dt} = F_x\big(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t\big) \\ m\dfrac{dv_y}{dt} = F_y\big(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t\big) \\ m\dfrac{dv_z}{dt} = F_z\big(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t\big) \end{cases}, \quad \begin{cases} \dfrac{dx}{dt} = v_x(t) \\ \dfrac{dy}{dt} = v_y(t) \\ \dfrac{dz}{dt} = v_z(t) \end{cases}$$

× N (number of particles)

... 6N ordinary differential equations + Maxwell's equations

Fluid approach

(in the case of magnetohydrodynamics)

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \boldsymbol{v}) = 0$$ ... for ρ

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \bullet \nabla \boldsymbol{v}\right) = -\nabla p + \frac{1}{\mu_0}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \boldsymbol{F}$$ ... for v_x, v_y, v_z

$$\frac{\partial}{\partial t}\left(\frac{p}{\gamma - 1}\right) + \nabla \bullet \left(\frac{p}{\gamma - 1}\boldsymbol{v}\right) = -p\nabla \bullet \boldsymbol{v} + \nabla \bullet (\kappa_c \nabla T) + \frac{j^2}{\sigma}$$ ... for P

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left(\boldsymbol{v} \times \boldsymbol{B} - \eta_{diff} \nabla \times \boldsymbol{B}\right)$$ ... for B_x, B_y, B_z

... 8 partial differential equations
(+ equation of state)

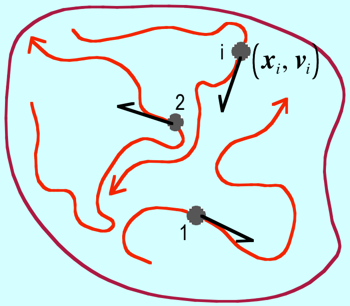
Because of local charge neutrality $\rho_c \sim 0$, Coulombic electric field does not exist globally (but electric field associated with time-varying magnetic field globally exists).

Since electric current globally exists while keeping the local charge neutrality, magnetic field globally exists.

Three types of dynamic systems (depend on total number of particles N)

I. Small N system (discrete system) => Mechanical equation

fundamental object... **particle** (**kinetic approach**)



We can distinguish each particle from others.

Focus on **Position** and **Velocity** of **every particle:** $\boldsymbol{x}_i(t),\ \boldsymbol{v}_i(t)$

Solve **Mechanical equation** with one independent variable **(time)** for all particles.

$$\begin{cases} \dfrac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i \\ m_i \dfrac{d\boldsymbol{v}_i}{dt} = \boldsymbol{F}_i \end{cases} \quad i = 1, 2, 3, \ldots, N$$

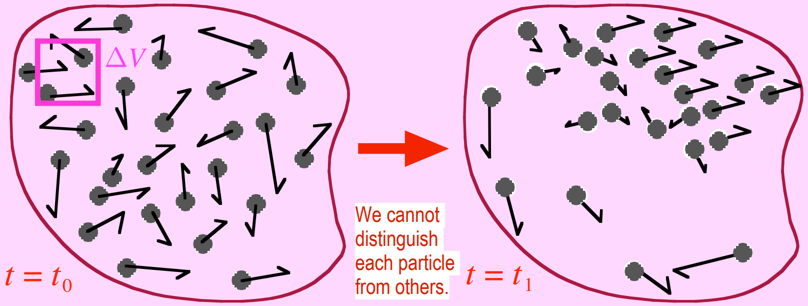
=> Derive **evolutionary path** of **every particle**

... the most complete solution

II. Intermediate N system (discrete system*) => Boltzmann's equation

***Sufficient number of particles exist** in **every local region** ΔV **to determine particle distribution everywhere**.

fundamental object... **particle** (**kinetic approach**)



Focus on Partice Distribution : $f(\boldsymbol{x}, \boldsymbol{v}, t)$

(Give up deriving the evolutioary path of every particle

=> derive **evolution** of **particle distribution**)

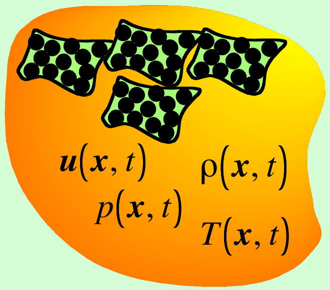
Solve Boltzmann's equation with seven independent variables (position, velocity, time) for particle distribution.

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \bullet \frac{\partial f}{\partial \boldsymbol{x}} + \frac{\boldsymbol{F}}{m} \bullet \frac{\partial f}{\partial \boldsymbol{v}} = \left(\frac{\delta f}{\delta t}\right)_C$$

III. Large N system (continuous system*) => Fluid dynamics equations

***Sufficient number of particles exist** in **each fluid element** and **keep staying in** it.

fundamental object... **fluid element** (**fluid approach**)



Use **continuum approximation:** fluid elements fill up the entire volume of system => particle-based field: $\rho, \boldsymbol{u}, P, T$

Derive **Thermal & Dynamic evolution** of **every fluid element:** $\rho(\boldsymbol{x}, t)$, $\boldsymbol{u}(\boldsymbol{x}, t)$, $T(\boldsymbol{x}, t)$, $p(\boldsymbol{x}, t)$

Take average of Boltzmann's equation using **Maxwellian distribution function** to derive **Fluid dynamics equations.**

Solve **Fluid dynamics equations** with four independent variables **(position, time)** for all fluid elements.