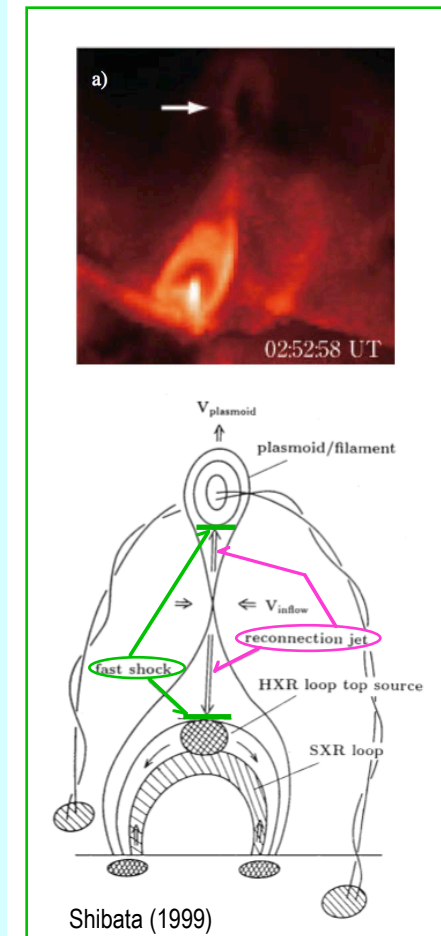
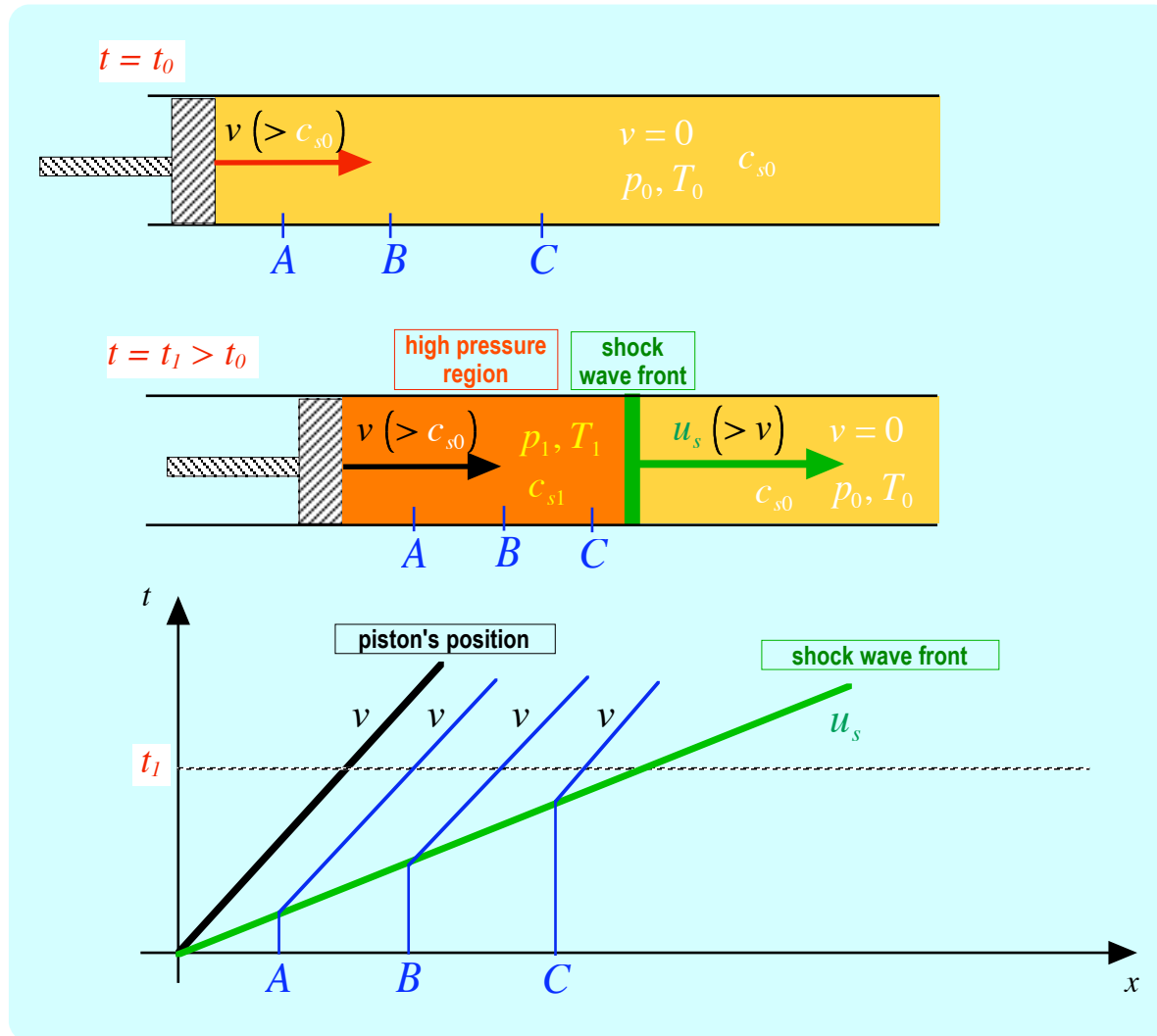


3) Kinetic energy is deposited: **piston-driven shock, jet**

associated with a **highly dynamic** event



4) *Spontaneous enhancement of a small-amplitude wave via nonlinear effect*

associated with a **less dynamic** event

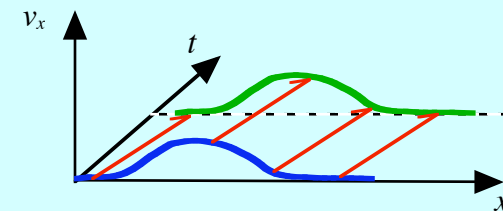
$$\rho \left(\frac{\partial}{\partial t} \mathbf{v} + \underbrace{\mathbf{v} \cdot \nabla \mathbf{v}}_{\text{nonlinear term}} \right) = \dots$$

Consider an 1-dimensional case:

$$\frac{\partial}{\partial t} v_x + \boxed{v_x} \frac{\partial v_x}{\partial x} = 0$$

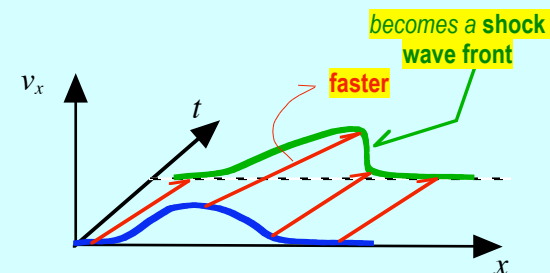
If this is approximated as a constant value, the profile of v_x is **conserved** (linear wave).

The profile propagates at the **same** speed.



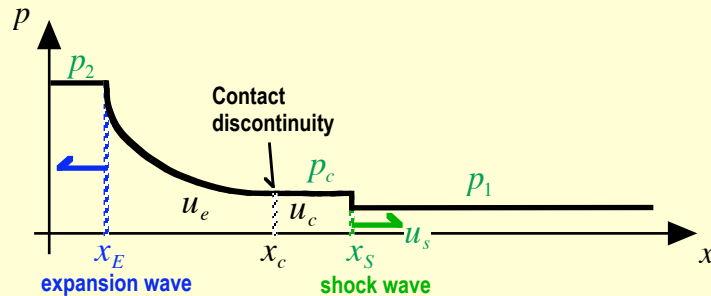
In fact, the propagation speed depends on the value of v_x , so the profile of v_x is **deformed and steepened** (nonlinear effect).

The profile propagates at **different** speeds.



Analytic solutions of shock tube, blast wave, and piston-driven shock

shock tube



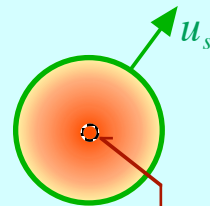
$$u_s = \frac{\gamma_1 + 1}{4} u_c + \sqrt{\frac{1}{16} (\gamma_1 + 1)^2 u_c^2 + c_1^2}$$

$$u_c = c_{s1} \left(\frac{p_c}{p_1} - 1 \right) \sqrt{\frac{2/\gamma_1}{(\gamma_1 + 1) p_c/p_1 + (\gamma_1 - 1)}}$$

$$\frac{p_2}{p_1} = \frac{p_c}{p_1} \left[1 - \frac{(\gamma_2 - 1) c_{s1}/c_{s2} (p_c/p_1 - 1)}{\sqrt{2\gamma_1} \sqrt{2\gamma_1 + (\gamma_1 + 1) (p_c/p_1 - 1)}} \right]^{-2 \frac{\gamma_2}{\gamma_2 - 1}}$$

(From *Elements of Gas Dynamics*)

blast wave



Thermal energy $\mathcal{E}$ is initially deposited here

$$u_s \equiv \frac{dr_{sh}}{dt} = \frac{2}{5} \xi_0 \left(\frac{\mathcal{E}}{\rho_1 t^3} \right)^{\frac{1}{5}}, \quad \xi_0 \sim o(1)$$

Sedov's solution

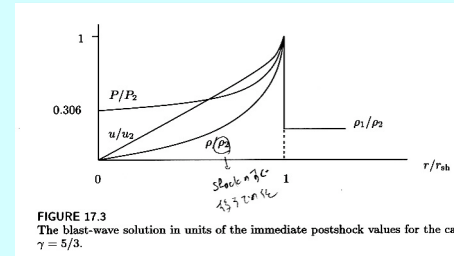
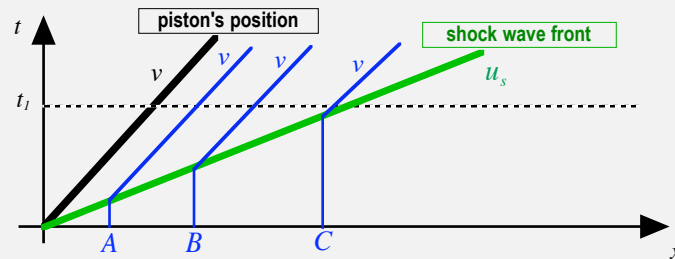


FIGURE 17.3
The blast-wave solution in units of the immediate postshock values for the case $\gamma = 5/3$.

(From *Gas Dynamics*)

piston-driven shock



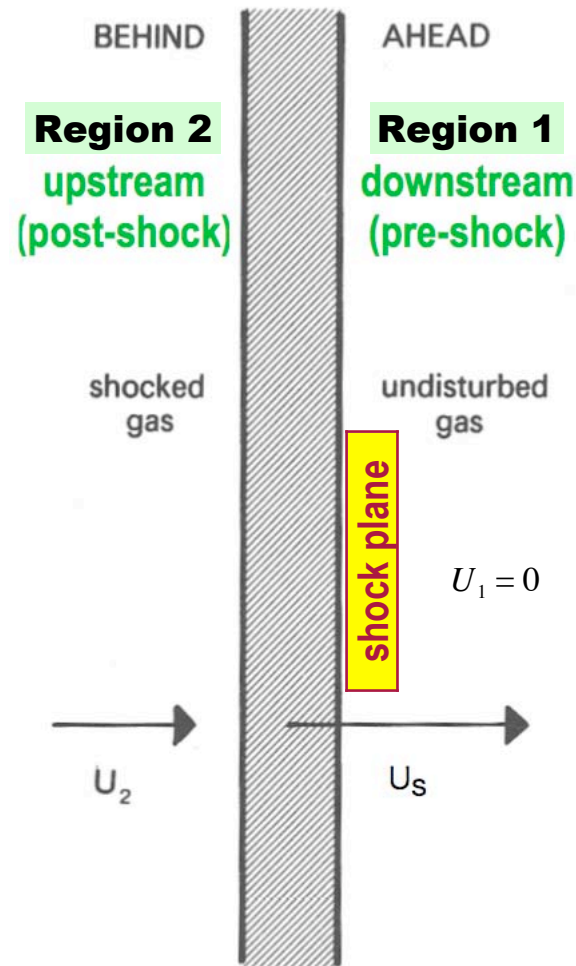
$$u_s = \frac{\gamma + 1}{4} v + \sqrt{\frac{1}{16} (\gamma + 1)^2 v^2 + c_s^2}$$

(From *Fluid Mechanics*)

Hydrodynamic shock wave

Perpendicular shock

... a **plane shock wave** propagating at a **constant speed** in the direction **perpendicular to the shock plane**



(From *Solar MHD*)

Converts rest frame to shock frame...

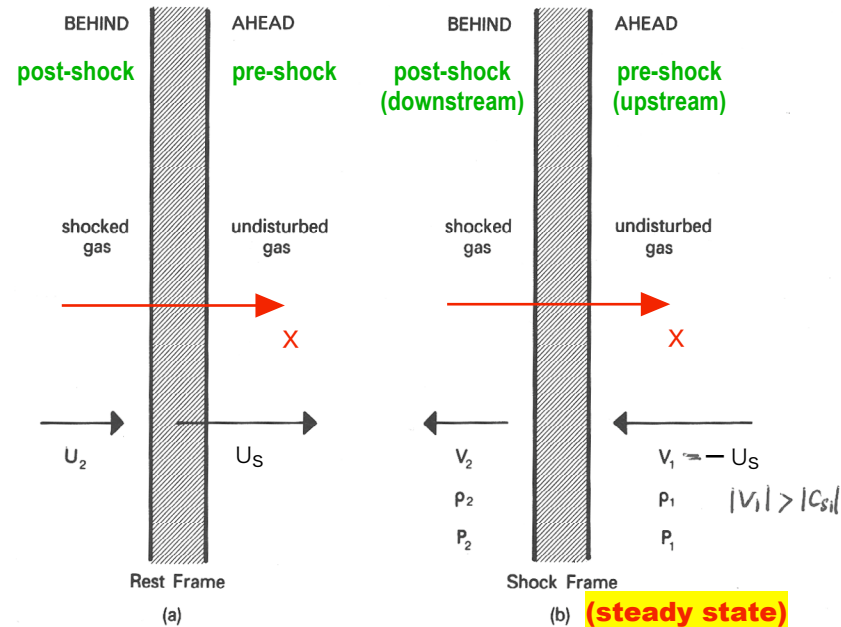


Fig. 5.3. The notation for a plane hydrodynamic shock wave moving to the right with speed u into a gas at rest. Properties ahead of the shock are denoted by 1 and those behind by 2.

(From *Solar MHD*)

Relation between the **rest frame** and **shock frame (steady state)**:

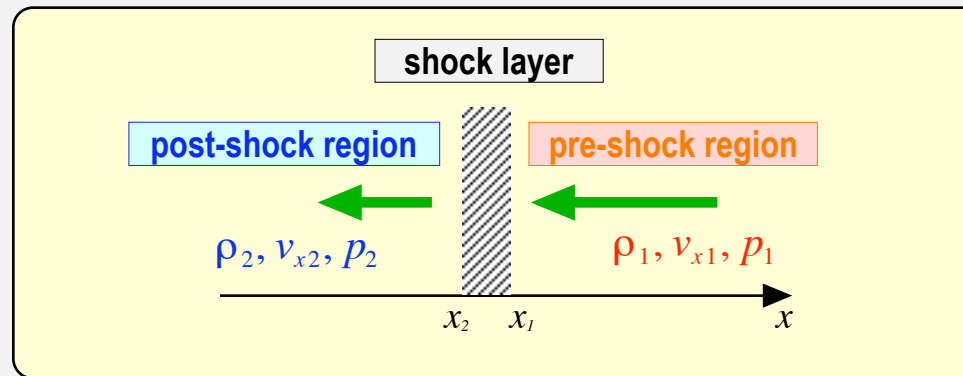
flow velocity in the pre-shock region is given by $v_{x1} = -U_s$

flow velocity in the post-shock region is given by $v_{x2} = v_{x1} + U_2 = -U_s + U_2$

In the shock frame, we consider three conservation laws (mass, momentum, energy) across a shock layer.

Mass conservation (1D, steady condition):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) = 0 \rightarrow \frac{d}{dx} (\rho v_x) = 0$$



$$\int_{x_2}^{x_1} \frac{d}{dx} (\rho v_x) dx = [\rho v_x]_{x_2}^{x_1} = \rho_1 v_{x1} - \rho_2 v_{x2} = 0$$

$$\rightarrow \rho_1 v_{x1} = \rho_2 v_{x2}$$