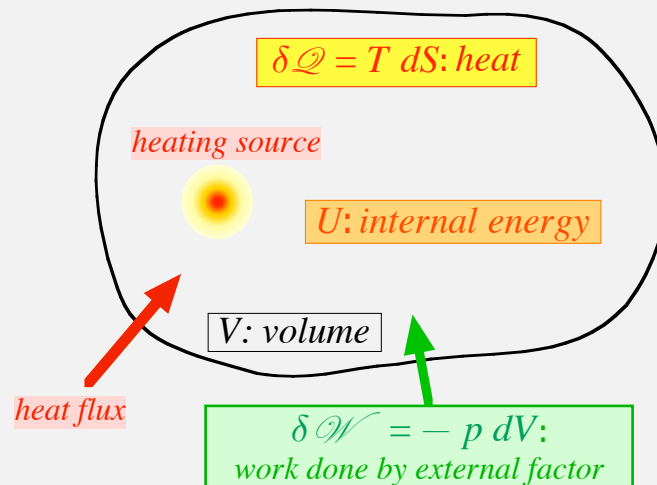


Equation III... internal energy equation

The first law of thermodynamics:

$$\delta Q + \delta W = dU$$

$$\Rightarrow T dS - p dV = dU$$



δQ ... heat

δW ... work

$\Rightarrow$ process-dependent quantities

S ... entropy

V ... volume

p ... gas pressure

U ... internal energy

$\Rightarrow$ state-dependent quantities



$$\delta Q_{\text{process 1}} \neq \delta Q_{\text{process 2}}$$

$$dV_{A \rightarrow B} = V_B - V_A$$

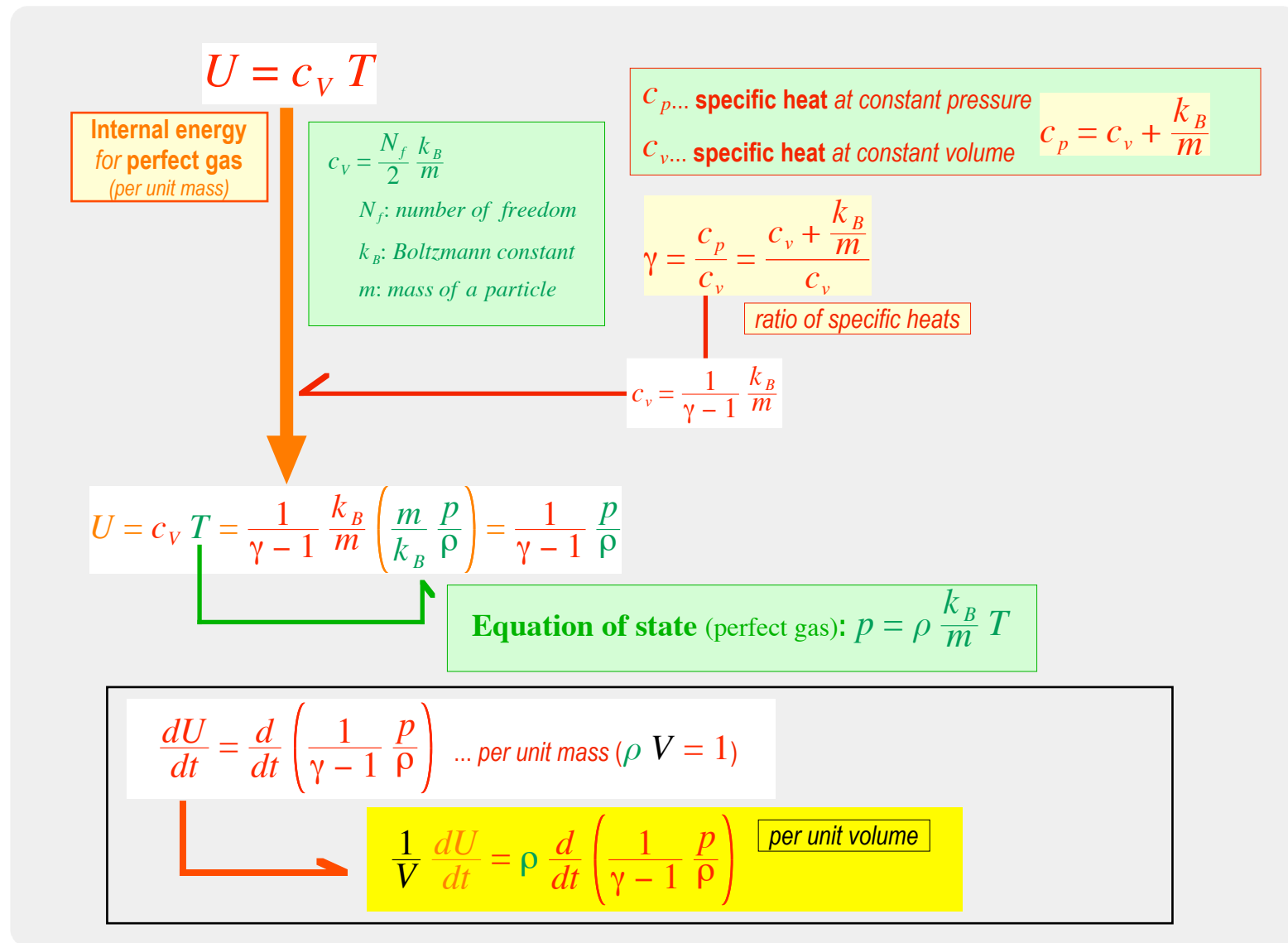
uniquely determined from the states A & B

Internal energy equation (per unit volume):

$$\frac{1}{V} T \frac{dS}{dt} - \frac{1}{V} p \frac{dV}{dt} = \frac{1}{V} \frac{dU}{dt}$$

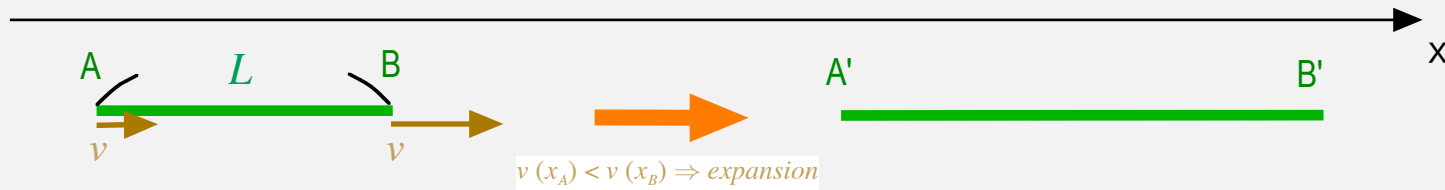
We then rewrite this equation using MHD quantities.

$\frac{1}{V} \frac{dU}{dt}$... rate of internal energy change



$$\frac{1}{V} \frac{dV}{dt} \dots \text{expansion rate}$$

1-dimensional case

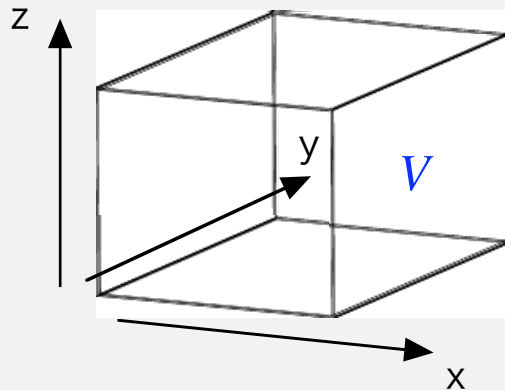


$$\text{Expansion rate} \equiv \frac{1}{L} \frac{dL}{dt} \Rightarrow \frac{1}{x_B - x_A} \frac{d}{dt} (x_B - x_A) \Rightarrow \frac{1}{\Delta x} [v_x(x + \Delta x) - v_x(x)] \xrightarrow{\Delta x \rightarrow 0} \frac{\partial v_x}{\partial x}$$

$x_A = x$

$x_B = x + \Delta x$

3-dimensional case



per unit volume

$$\text{Expansion rate} \equiv \frac{1}{V} \frac{dV}{dt} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \equiv \nabla \cdot \mathbf{v}$$

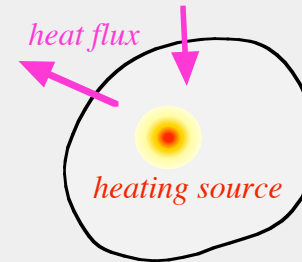
$$\frac{1}{V} T \frac{dS}{dt} \dots \text{heating rate}$$

heating = **net heat influx** (*external origin*) + **heating source** (*internal origin*)

"net influx" $\Rightarrow -\nabla \cdot$

$$\frac{1}{V} T \frac{dS}{dt} = -\nabla \cdot F_c + \eta j^2 + H$$

per unit volume



thermal conduction flux

$$F_c = -\kappa_c \nabla T$$

κ_c thermal conductivity

Joule heating source
(from Ohm's law)

Other heating sources
viscous heating
...

$$\frac{1}{V} T \frac{dS}{dt} = 0 \dots \text{adiabatic process}$$

There is neither **net heat influx** nor **heating source**. **Entropy** $dS = \frac{\delta Q}{T}$ is conserved.

$$\frac{1}{V} T \frac{dS}{dt} - \frac{1}{V} p \frac{dV}{dt} = \frac{1}{V} \frac{dU}{dt} \quad \text{per unit volume}$$

$$\rho \frac{d}{dt} \left(\frac{1}{\gamma - 1} \frac{p}{\rho} \right)$$

$$p \nabla \cdot \mathbf{v}$$

$$-\nabla \cdot \mathbf{F}_c + \eta j^2 + H$$

Internal energy eq. in MHD

$$\rho \frac{d}{dt} \left(\frac{1}{\gamma - 1} \frac{p}{\rho} \right) = -p \nabla \cdot \mathbf{v} - \nabla \cdot \mathbf{F}_c + \eta j^2 + H - \nabla \cdot \mathbf{F}_R$$

compression (source; reversible) conduction (flux) heating (source; irreversible) radiation (flux)

e.g. Radiative transfer theory

$$\Rightarrow \rho \frac{\partial}{\partial t} \left(\frac{1}{\gamma - 1} \frac{p}{\rho} \right) + \rho \mathbf{v} \cdot \nabla \left(\frac{1}{\gamma - 1} \frac{p}{\rho} \right) = \frac{\partial}{\partial t} \left(\frac{p}{\gamma - 1} \right) + \nabla \cdot \left(\frac{p}{\gamma - 1} \mathbf{v} \right)$$

convection (flux)