

Nonlinear term of flow velocity: $(\mathbf{v} \cdot \nabla) \mathbf{v}$

$$\left(\rho^p v_k^p \right) \frac{\partial}{\partial x_k} v_i^p + \left(\rho^e v_k^e \right) \frac{\partial}{\partial x_k} v_i^e = n M \left(v_k^p \frac{\partial}{\partial x_k} v_i^p + \frac{m}{M} v_k^e \frac{\partial}{\partial x_k} v_i^e \right) = n M \left[v_k^p \frac{\partial}{\partial x_k} v_i^p \left(\frac{m}{M} \right)^0 + O \left(\left(\frac{m}{M} \right)^1 \right) \right]$$

1st-order term of $\frac{m}{M}$

$$\left(\rho^{MHD} v_k^{MHD} \right) \frac{\partial}{\partial x_k} v_i^{MHD} = n (M + m) \frac{M v_k^p + m v_k^e}{M + m} \frac{\partial}{\partial x_k} \left(\frac{M v_i^p + m v_i^e}{M + m} \right)$$

$$= n M \left(v_k^p + \frac{m}{M} v_k^e \right) \frac{\partial}{\partial x_k} \left(\frac{v_i^p + \frac{m}{M} v_i^e}{1 + \frac{m}{M}} \right) = n M v_k^p \frac{\partial}{\partial x_k} v_i^p \left(\frac{m}{M} \right)^0 + O \left(\left(\frac{m}{M} \right)^1, \left(\frac{m}{M} \right)^2, \dots \right)$$

1st-order and even higher-order terms of $\frac{m}{M}$

comparison

$$\left(\rho^p v_k^p \right) \frac{\partial}{\partial x_k} v_i^p + \left(\rho^e v_k^e \right) \frac{\partial}{\partial x_k} v_i^e \approx \left(\rho^{MHD} v_k^{MHD} \right) \frac{\partial}{\partial x_k} v_i^{MHD}$$

based on the 0th-order approximation of $\frac{m}{M}$

$$\rho^{MHD} \left[\frac{\partial}{\partial t} \mathbf{v}^{MHD} + \left(\mathbf{v}^{MHD} \cdot \nabla \right) \mathbf{v}^{MHD} \right] = \mathbf{j}^{MHD} \times \mathbf{B} - \nabla P^{MHD}$$

Momentum eq. in MHD

Ohm's law

Subtract **electron's momentum equation** $\times M$ from **proton's momentum equation** $\times m$

(subtraction of $(\mathbf{v} \cdot \nabla) \mathbf{v}$ terms is neglected $\Leftarrow$ large spatial scale, long time scale)

$$n \frac{M m}{e} \frac{\partial}{\partial t} \left(\frac{\mathbf{j}^{MHD}}{n} \right) = e \rho^{MHD} \mathbf{E} + (M + m) \mathbf{f}_c^{p-e} - m \nabla P^p + M \nabla P^e + n e (m \mathbf{v}^p + M \mathbf{v}^e) \times \mathbf{B}$$

$$\mathbf{f}_c^{p-e} = -n e \eta \mathbf{j}^{MHD} \quad -\mathbf{f}_c^{p-e} \propto \mathbf{j}^{MHD} \text{ (both } \propto \mathbf{v}^p - \mathbf{v}^e \text{)}$$

$$\eta = \frac{M}{n e^2 \tau_c^{p-e}} = \frac{m}{n e^2 \tau_c^{e-p}} : \text{resistivity}$$

$$\eta_{diff} \equiv \frac{1}{\mu_0} \eta = \frac{c^2 \tau_p^2}{\tau_c^{e-p}} : \text{magnetic diffusivity}$$

MKS

$$\begin{aligned} m \mathbf{v}^p + M \mathbf{v}^e &= M \mathbf{v}^p + m \mathbf{v}^e + M (\mathbf{v}^e - \mathbf{v}^p) + m (\mathbf{v}^p - \mathbf{v}^e) \\ &= \frac{\rho^{MHD}}{n} \mathbf{v}^{MHD} - (M - m) \frac{\mathbf{j}^{MHD}}{n e} \end{aligned}$$

$$\mathbf{E} = -\mathbf{v}^{MHD} \times \mathbf{B} + \eta \mathbf{j}^{MHD} + \frac{M}{e \rho^{MHD}} \left[n \frac{m}{e} \frac{\partial}{\partial t} \left(\frac{\mathbf{j}^{MHD}}{n} \right) + \left(1 - \frac{m}{M} \right) \mathbf{j}^{MHD} \times \mathbf{B} + \nabla \left(\frac{m}{M} P^p - P^e \right) \right]$$

approximation

assumption

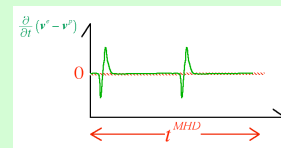
approximation

approximation

assumption & approximation

Slow evolution: $\frac{\partial}{\partial t} \left(\frac{\mathbf{j}^{MHD}}{n} \right) \sim 0 \Leftarrow \frac{\partial}{\partial t} (\mathbf{v}^p - \mathbf{v}^e) \sim 0$ on MHD scale

Only $\left(\frac{m}{M} \right)^0$ -term is taken into account.



$$\mathbf{E}^{MHD} = -\mathbf{v}^{MHD} \times \mathbf{B} + \eta \mathbf{j}^{MHD} + \frac{1}{e n} \left[\mathbf{j}^{MHD} \times \mathbf{B} - \nabla P^e \right]$$

Ohm's law in MHD

(determines electric field from flow velocity & current density)

Derivative in mathematics... $\Delta t \rightarrow 0, \Delta x \rightarrow 0$

Derivative in physics... based on typical scale
 $\Delta t \rightarrow t^{MHD}, \Delta x \rightarrow l^{MHD}$

assumption

Electron's inertia is small enough to remove nonuniformity of electron's pressure on MHD scale.

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