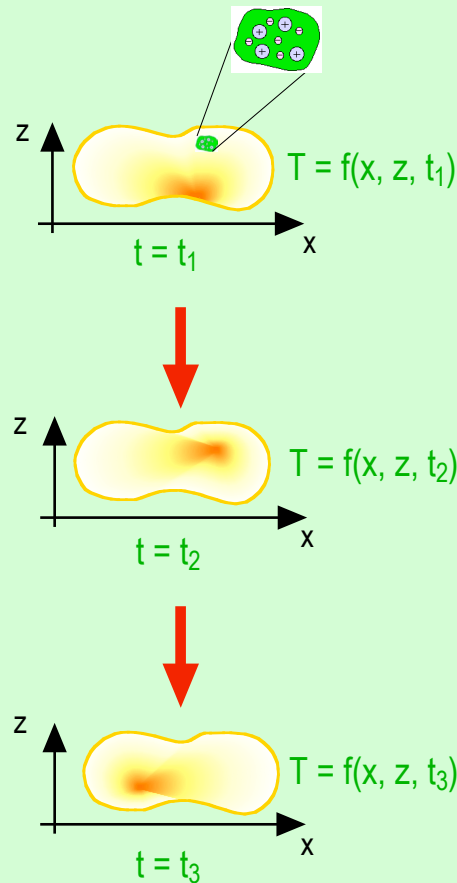


Fluid approach... Focus on *density, flow velocity, pressure, temperature* of each fluid element in *space-time* (these are *statistically averaged quantities* derived from *position & velocity distributions of particles*)

=> **Fluid dynamics equations**



In the **fluid approach**, we consider the *physical state* of each fluid element represented by *statistically averaged quantities* such as *density, flow velocity, pressure, and temperature*. For example, *temperature* changes with *position* and *time*, so it is expressed as a function of *position* and *time*:

$$T = f(x, z, t)$$

Temperature field

continuously distributed in space-time

Particles do not fill up the space, whereas fluid elements do that.

An equation for the *temperature field* is a differential equation where *position* and *time* are independent variables (**partial differential equation, PDE**).

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

Kinetic approach

(in the case of mechanics)

$$\left\{ \begin{array}{l} m \frac{dv_x}{dt} = F_x(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t) \\ m \frac{dv_y}{dt} = F_y(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t) \\ m \frac{dv_z}{dt} = F_z(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t) \end{array} \right. , \quad \left\{ \begin{array}{l} \frac{dx}{dt} = v_x(t) \\ \frac{dy}{dt} = v_y(t) \\ \frac{dz}{dt} = v_z(t) \end{array} \right. \quad \times N \text{ (number of particles)}$$

... 6N ordinary differential equations
+ Maxwell's equations

Fluid approach

(in the case of magnetohydrodynamics)

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \quad \dots \text{ for } \rho \\ \rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) &= -\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} + \mathbf{F} \quad \dots \text{ for } v_x, v_y, v_z \\ \frac{\partial}{\partial t} \left(\frac{p}{\gamma - 1} \right) + \nabla \cdot \left(\frac{p}{\gamma - 1} \mathbf{v} \right) &= -p \nabla \cdot \mathbf{v} + \nabla \cdot (\kappa_e \nabla T) + \frac{j^2}{\sigma} \quad \dots \text{ for } P \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B} - \eta_{diff} \nabla \times \mathbf{B}) \quad \dots \text{ for } B_x, B_y, B_z \end{aligned}$$

... 8 partial differential equations
(+ equation of state)

Because of **local charge neutrality** $\rho_c \sim 0$, **Coulombic electric field does not exist globally** (but **electric field associated with time-varying magnetic field may globally exist**).

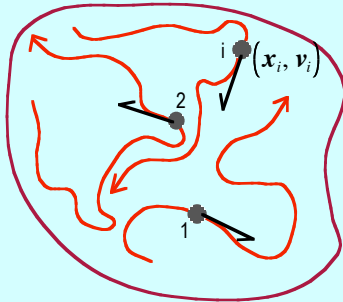
Since **electric current globally exists while keeping the local charge neutrality**, **magnetic field globally exists**.

$$\begin{aligned} E_{coulomb}(\mathbf{r}) &= \iiint_{V_0} \frac{\rho_c(\mathbf{r}') (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV' \\ \mathbf{B}(\mathbf{r}) &= \frac{1}{c^2} \iiint_{V_0} \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV' \end{aligned}$$

Three types of dynamic systems (depend on *total number of particles N*)

I. Small N system (discrete system) => Mechanical equation

fundamental object... **particle**
(kinetic approach)



We can distinguish each particle from others.

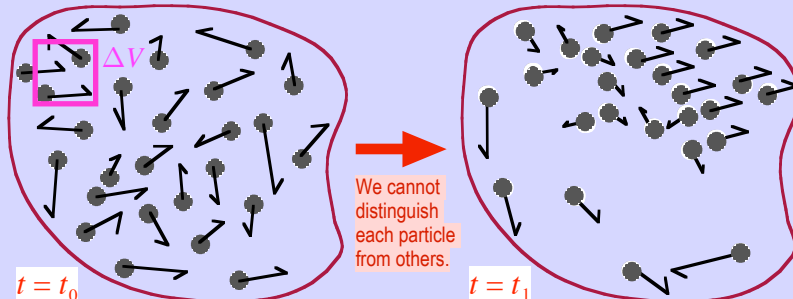
- Focus on **Position** and **Velocity** of **every particle**: $\mathbf{x}_i(t)$, $\mathbf{v}_i(t)$
- Solve **Mechanical equation** with one independent variable (**time**) for all particles.

$$\begin{cases} \frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i \\ m_i \frac{d\mathbf{v}_i}{dt} = \mathbf{F}_i \end{cases} \quad i = 1, 2, 3, \dots, N \Rightarrow \text{Derive the } \mathbf{evolutionary path of every particle} \\ \mathbf{\dots the most complete solution}$$

II. Intermediate N system (discrete system*) => Boltzmann's equation

fundamental object... **particle**
(kinetic approach)

*Sufficient number of particles exist in every local region ΔV to give particle distribution everywhere.



We cannot distinguish each particle from others.

- Focus on **Particle Distribution**: $f(\mathbf{x}, \mathbf{v}, t)$
(Give up deriving the evolutionary path of every particle
 $\Rightarrow$ Derive the **evolution of particle distribution**)

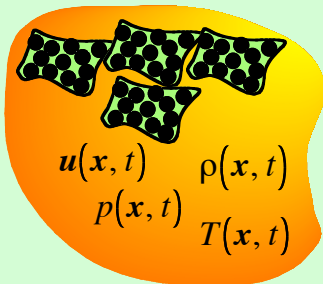
- Solve **Boltzmann's equation** with seven independent variables (**position, velocity, time**) for the particle distribution.

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\delta f}{\delta t} \right)_c$$

III. Large N system (continuous system**) => Fluid dynamics equations

fundamental object... **fluid element**
(fluid approach)

**Sufficient number of particles exist in each fluid element and keep staying in it.



- Use **continuum approximation**: fluid elements fill up the entire volume of system $\Rightarrow$ particle-based fields: $\rho, \mathbf{u}, P, T$
 $\Rightarrow$ Derive the **thermal & dynamic evolution of every fluid element**: $\rho(\mathbf{x}, t)$, $\mathbf{u}(\mathbf{x}, t)$, $T(\mathbf{x}, t)$, $p(\mathbf{x}, t)$
- Take average of **Boltzmann's equation** using **Maxwellian distribution function** to derive **Fluid dynamics equations**.
- Solve **Fluid dynamics equations** with four independent variables (**position, time**) for all fluid elements.

Summary of plasma, kinetic & fluid approaches

Plasma... 4th state of matter in a system where

a number of charged particles exist

these particles **behave collectively** ($\Rightarrow$ *local charge neutrality*)

Kinetic approach... more fundamental approach in the sense that

it is based on **real object** (particle)

it can be used **even for non-plasma state** (*each particle behaves independently*)

Fluid approach... less fundamental approach in the sense that

it is based on **virtual object** (fluid element)

it can be used **only for thermal state** (*velocity distribution of particles follows Maxwellian distribution*)