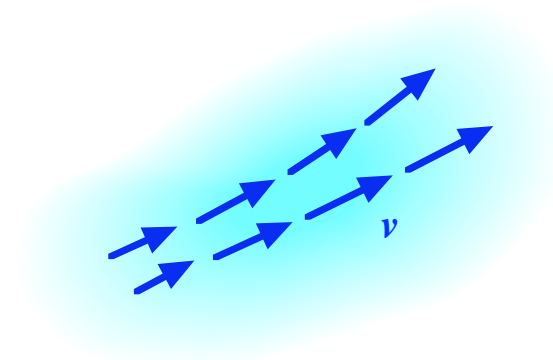


Irrotational and incompressible flow

Irrotational flow (potential flow)... $\nabla \times \mathbf{v} \equiv \text{rot } \mathbf{v} = \mathbf{0}$

$$\nabla \times \mathbf{v} \equiv \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \\ \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \end{pmatrix}$$



$$\mathbf{v} = -\nabla \Phi_v, \quad \Phi_v: \text{velocity potential} \Rightarrow \nabla \times \mathbf{v} = \mathbf{0}$$

Incompressible flow... $\nabla \cdot \mathbf{v} \equiv \text{div } \mathbf{v} = 0$

$$\mathbf{v} = -\nabla \Phi_v$$

$$\nabla \cdot \mathbf{v} = 0$$

$$\nabla \cdot (-\nabla \Phi_v) = -\nabla^2 \Phi_v = 0$$

Laplace equation

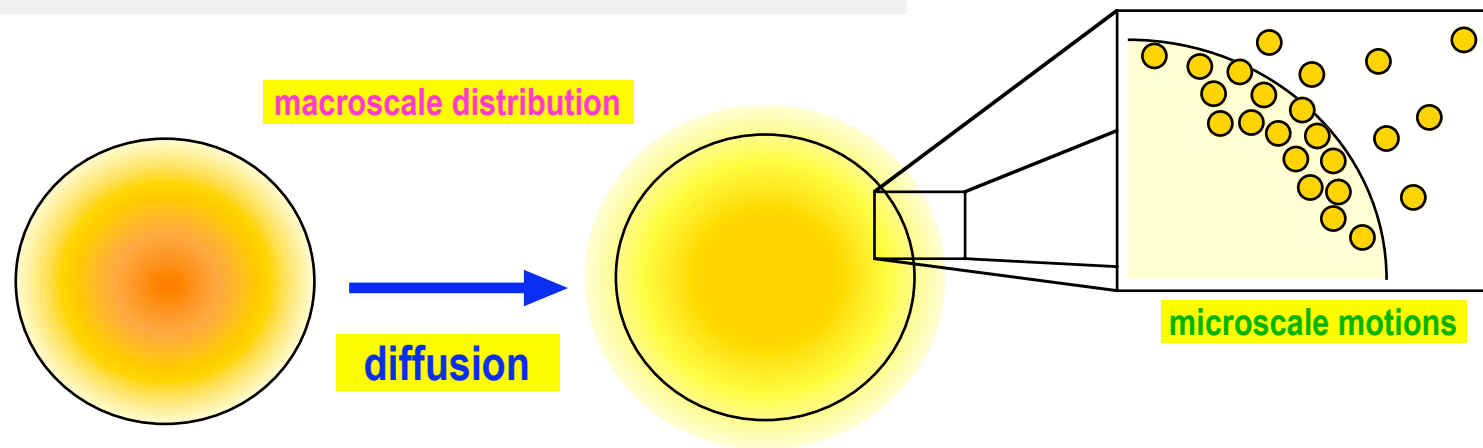
Diffusion equation

What is diffusion?

A **time-dependent** process

caused by **microscale motions of particles**

smoothes **macroscale distribution of particles**

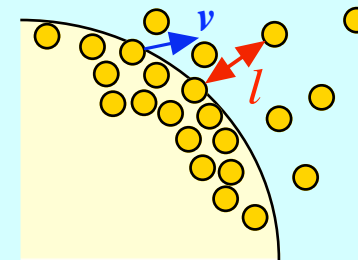


Diffusion is characterized by

$$\text{length } l \times \text{velocity } v = l \times \frac{l}{\tau_c} = \frac{l^2}{\tau_c}$$

l ... mean free path/inertial length

τ_c ... collision time



Thermal conduction... typical diffusion process

Temperature flux: $\mathbf{F}_T = -\alpha \nabla T$

... from a high temperature region to low temperature region

α : thermal diffusivity... dimension is given by $v \times l$ (thermal speed $\times$ mean free path)

Temperature change rate: $\frac{\partial T}{\partial t}$

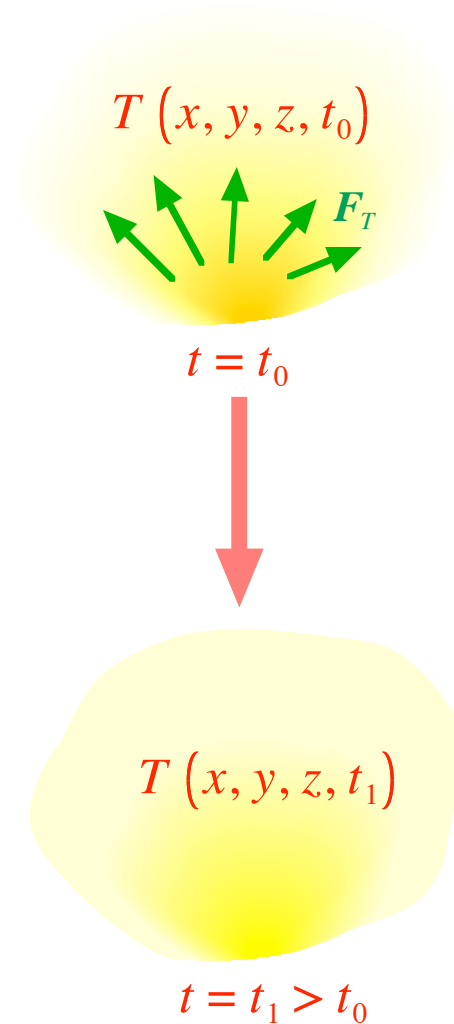
= **net influx** ($\equiv$ influx - outflux): $-\nabla \cdot \mathbf{F}_T$

$$\nabla \cdot \mathbf{F}_T \equiv \frac{\partial F_{Tx}}{\partial x} + \frac{\partial F_{Ty}}{\partial y} + \frac{\partial F_{Tz}}{\partial z} \dots \text{divergence of } \mathbf{F}_T$$

$$\frac{\partial T(x, y, z, t)}{\partial t} = \alpha \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) T(x, y, z, t)$$

Time-dependent part... **1st-order partial derivative**

Space-dependent part... **2nd-order partial derivative**



Evolution of a diffusion profile

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \Rightarrow \frac{\Delta T}{\tau_{dif}} \sim \alpha \frac{\Delta T}{l_{dif}^2}$$

order estimate

$$\left(\frac{\partial}{\partial t} \sim \frac{1}{\tau_{dif}}, \frac{\partial}{\partial x} \sim \frac{1}{l_{dif}} \right)$$

