

3. Numerical simulation is based on **partial differential equations (PDEs)** that describe temporal & spatial variations of physical quantities such as density, velocity, pressure, and magnetic field.

e.g.)

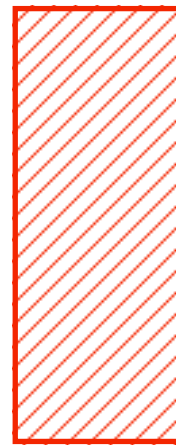
Magnetohydrodynamic equations... a set of PDEs

$$\frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \mathbf{v})$$

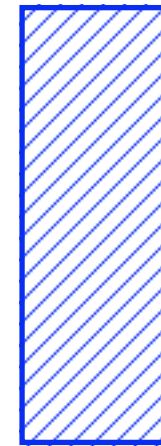
$$\frac{\partial \mathbf{v}}{\partial t} = - (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{- \nabla p + \mathbf{j} \times \mathbf{B}}{\rho}$$

$$\frac{\partial p}{\partial t} = - (\mathbf{v} \cdot \nabla) p - \gamma p \nabla \cdot \mathbf{v}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$



=



Time-dependent part Space-dependent part

The set of PDEs describe temporal & spatial variations of physical quantities $\rho, \mathbf{v}, P, \mathbf{B}, \mathbf{j}$, which are given by functions of space (x, y, z) and time (t) .

Why do we need numerical simulation?

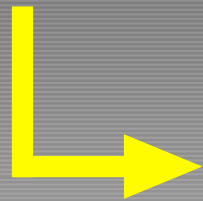
Basic equations given by a set of PDEs are **complex**;
they are **coupled with each other**.



Solving these equations **analytically**
to obtain an **exact solution** is difficult.

$$\begin{aligned}\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\ \frac{\partial \mathbf{v}}{\partial t} &= -(\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{-\nabla p + \mathbf{j} \times \mathbf{B}}{\rho} \\ \frac{\partial p}{\partial t} &= -(\mathbf{v} \cdot \nabla) p - \gamma p \nabla \cdot \mathbf{v} \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B})\end{aligned}$$

$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$



Therefore, we **perform numerical simulation**
to obtain an **approximate solution**, which
helps us understand the **physical mechanism**
of a phenomenon evolving in time and space.

Examples

Numerical simulations performed for investigating solar dynamic phenomena

Solar flare

Explosive phenomenon in solar corona

Observation of solar flare

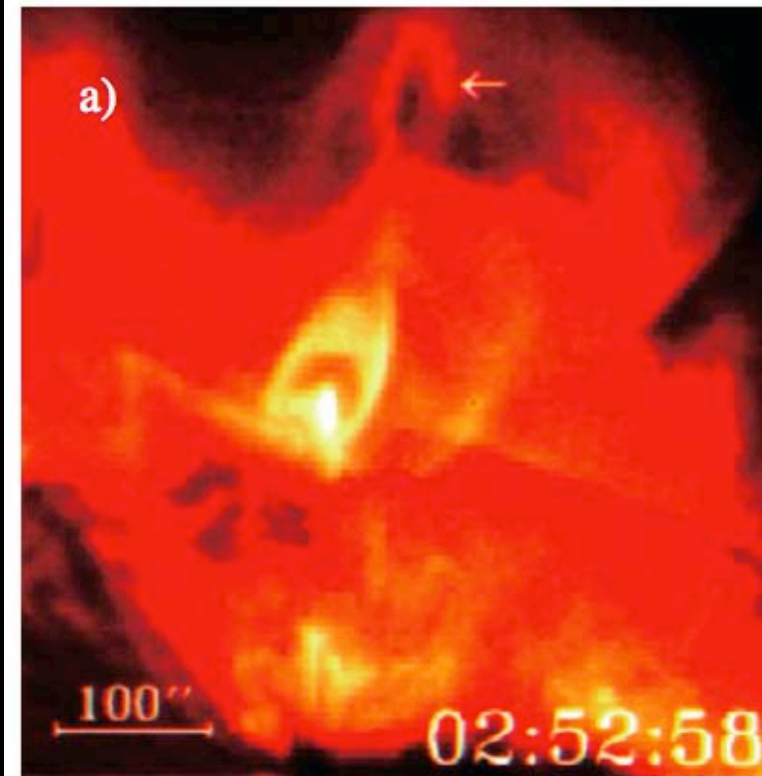


Full-disk image of a solar corona
(observed by *Yohkoh*)

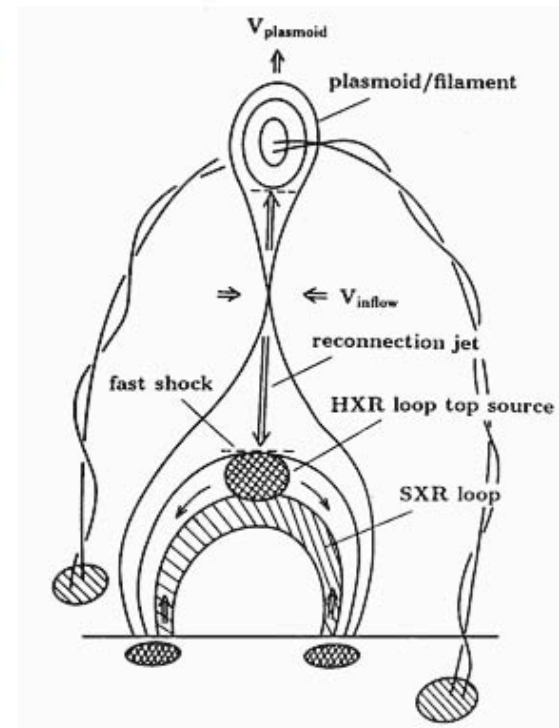


An X-class flare in an active region
(observed by *Hinode*)

Schematic model of solar flare



b)



Shibata et al. (1995)

Numerical simulation of solar flare



Shiota et al. (2005)

