Basic equations:

Mass conservation.. $\rho \, v_r \, A = const.$

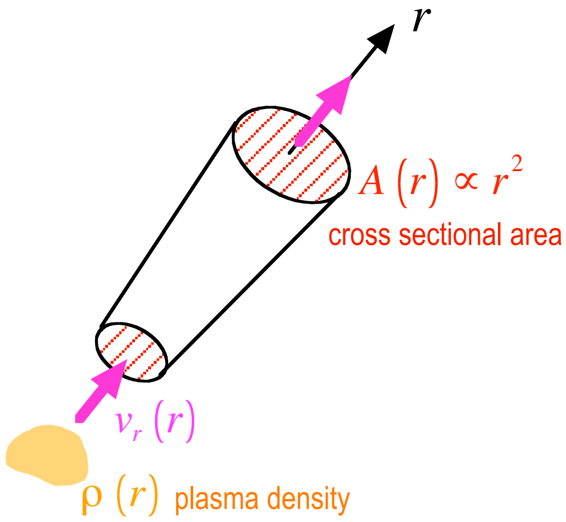
spherically symmetric => $A(r) \propto r^2$

Momentum equation...

$$\rho \, v_r \frac{\partial v_r}{\partial r} = -\frac{\partial p}{\partial r} - \frac{G M_\odot \rho}{r^2}$$

Energy equation... $T = const. \left(\frac{p}{\rho} = \frac{k_B T}{\bar{m}} = v_c^2 = const. \quad v_c = \sqrt{\frac{k_B T}{\bar{m}}} \right)$

isothermal sound speed



eliminate ρ and p

$$\left(v_r - \frac{v_c^2}{v_r} \right) \frac{\partial v_r}{\partial r} = \frac{2 v_c^2}{r} - \frac{G M_\odot}{r^2}$$

Lagrangian derivative vs. Eulerian derivative

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \left(\boldsymbol{v} \bullet \nabla \right)$$

$$d\,A\left(x, y, z, t\right) = \frac{\partial A}{\partial x}\,d\,x + \frac{\partial A}{\partial y}\,d\,y + \frac{\partial A}{\partial z}\,d\,z + \frac{\partial A}{\partial t}\,d\,t$$

total differential

$$\begin{aligned}
\therefore\ \frac{d\,A}{d\,t} &= \frac{\partial A}{\partial x}\frac{d\,x}{d\,t} + \frac{\partial A}{\partial y}\frac{d\,y}{d\,t} + \frac{\partial A}{\partial z}\frac{d\,z}{d\,t} + \frac{\partial A}{\partial t}\frac{d\,t}{d\,t} \\
&= \frac{\partial A}{\partial x}v_x + \frac{\partial A}{\partial y}v_y + \frac{\partial A}{\partial z}v_z + \frac{\partial A}{\partial t} \\
&= \left(v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} + v_z \frac{\partial}{\partial z} \right) A + \frac{\partial A}{\partial t} \\
&= \frac{\partial A}{\partial t} + \left(\boldsymbol{v} \bullet \nabla \right) A
\end{aligned}$$

Solutions of $\left(v_r - \frac{v_c^2}{v_r}\right)\frac{dv_r}{dr} = \frac{2\,v_c^2}{r} - \frac{G\,M_\odot}{r^2}$...

$\frac{\partial v_r}{\partial r} \Rightarrow \frac{dv_r}{dr}$

integrate

$$\int \left(v_r - \frac{v_c^2}{v_r}\right) dv_r = \int \left(\frac{2\,v_c^2}{r} - \frac{G\,M_\odot}{r^2}\right) dr$$

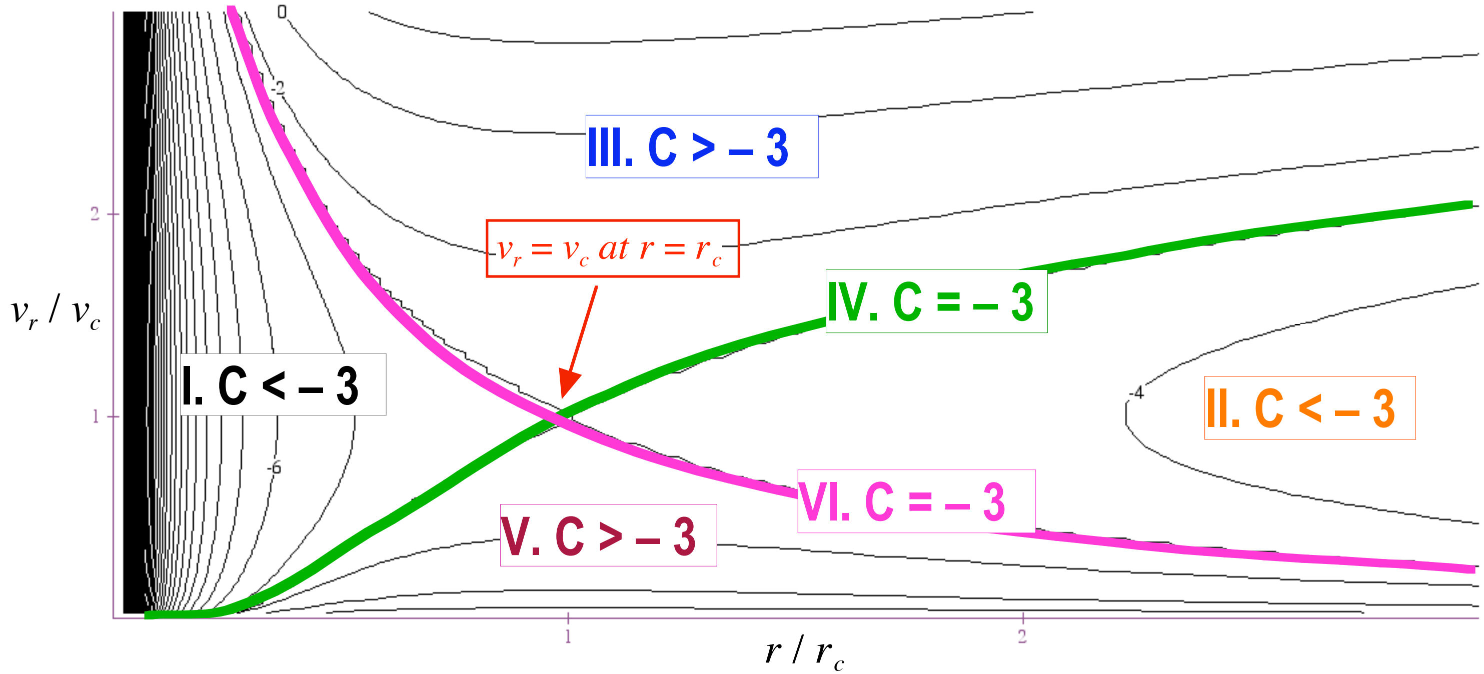
dimensionless form

$$\left(\frac{v_r}{v_c}\right)^2 - \ln\left(\frac{v_r}{v_c}\right)^2 = 4\ln\frac{r}{r_c} + 4\left(\frac{r}{r_c}\right)^{-1} + C$$

$r_c \equiv \frac{G\,M_\odot}{2\,v_c^2}$: **critical radius**

$C = constant$

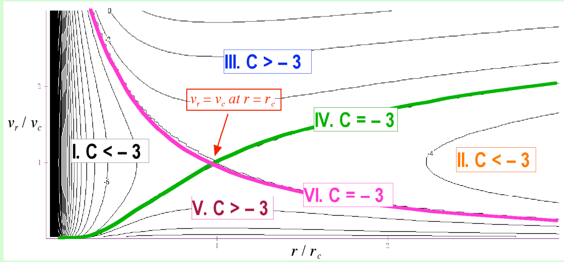
$C = -3$ when $(r, v_r) = (r_c, v_c)$.



Characteristics of the solutions...

I. C < – 3 **II. C < – 3**

These two solutions are **physically incorrect**
(although they are correct mathematically)
because flow velocity takes two different values at the same position.



III. C > – 3 **IV. C = – 3** **V. C > – 3** **VI. C = – 3**

These four solutions are physically correct. Which is appropriate for solar wind?

III. C > – 3 **VI. C = – 3** ... starts with a supersonic flow ($v_r > v_c$) at a solar surface
=> inconsistent with inner boundary condition

IV. C = – 3 ... starts with a subsonic flow ($v_r < v_c$) at a solar surface, and becomes supersonic
=> consistent with inner boundary condition

V. C > – 3 ... starts with a subsonic flow ($v_r < v_c$) at a solar surface, and remains subsonic
=> consistent with inner boundary condition

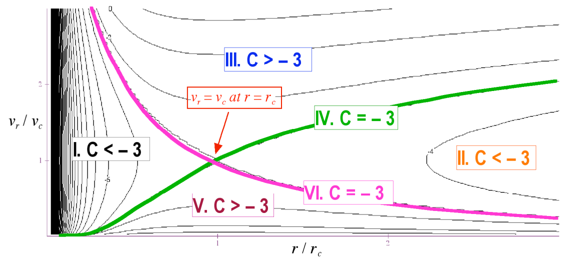
Outer boundary condition (solution IV vs. solution V)...

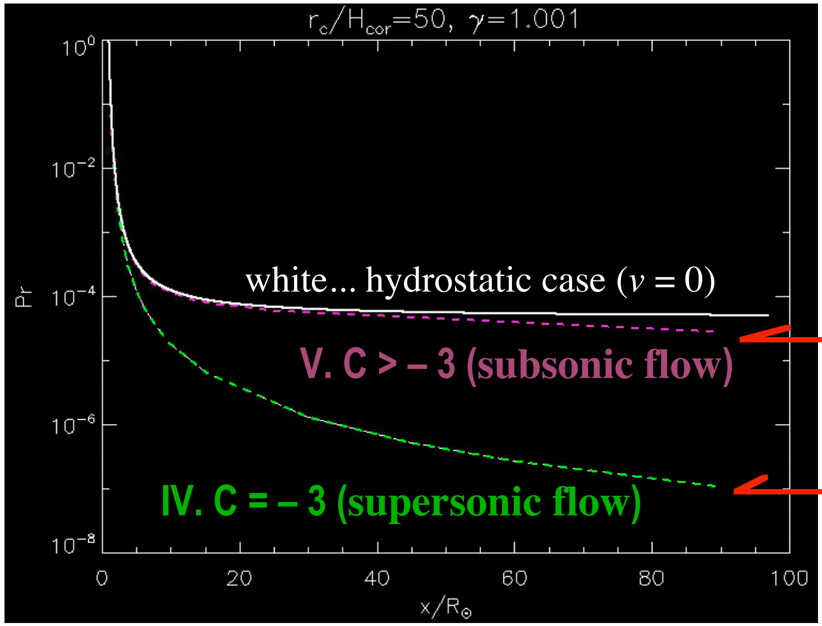
We focus on gas pressure distribution:

$$p(r) = \rho(r)\,\frac{k_B T}{\overline{m}} = \rho(r)\,v_c^2\,,\quad (v_c\ is\ constant)$$

$$\rho(r) = \frac{\rho(R_\odot)\,v_r(R_\odot)\,R_\odot^2}{v_r(r)\,r^2}$$

$v_r(r)$





$r_c \sim 3\ R_\odot\ with\ T \sim 1.5\times10^6\ K$

The value of gas pressure at outer boundary is different between IV (**supersonic case, low pressure**) and V (**subsonic case, high pressure**).

↓

Observation suggests a very low value of gas pressure at the outer boundary ($P^{interstellar} \sim 10^{-17}\ P^{solar\ surface}$), so we select solution IV (**supersonic case**).