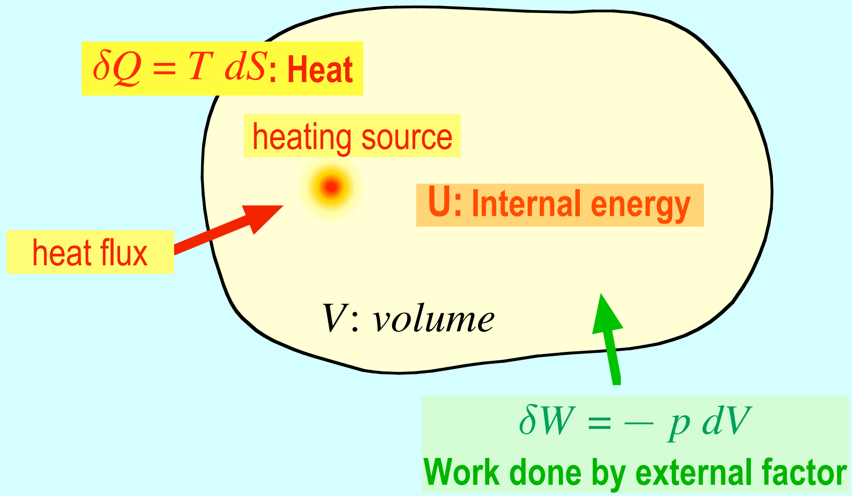
Equation III... internal energy equation

The first law of thermodynamics:

$$\delta Q + \delta W = dU \;\Rightarrow\; T\,dS - p\,dV = dU$$



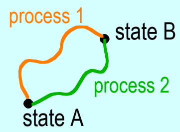
δQ... *heat*

δW... *work*

=> process-dependent quantities

S... *entropy*
V... *volume*
p... *gas pressure*
U... *internal energy*

=> state-dependent quantities



$\delta Q_{process\ 1} \neq \delta Q_{process\ 2}$

$dV_{A \to B} = V_B - V_A$
uniquely determined from states A & B

Internal energy equation (per unit volume):

$$\frac{1}{V}\,T\,\frac{dS}{dt} - \frac{1}{V}\,p\,\frac{dV}{dt} = \frac{1}{V}\,\frac{dU}{dt}$$

We then rewrite this equation using MHD quantities.

$\frac{1}{V}\frac{dU}{dt}$ ... rate of change in internal energy

$$U = c_V T$$

Internal energy per unit mass (perfect gas)

c_p... specific heat at constant pressure

c_v... specific heat at constant volume

$$c_p = c_v + \frac{k_B}{m}$$

$$\gamma = \frac{c_p}{c_v} = \frac{c_v + \frac{k_B}{m}}{c_v}$$

ratio of specific heat

$$c_v = \frac{1}{\gamma - 1}\frac{k_B}{m}$$

$$c_V = \frac{N_f}{2}\frac{k_B}{m}$$

N_f: number of freedom

k_B: Boltzmann constant

m: mass of a particle

$$U = c_V T = \frac{1}{\gamma - 1}\frac{k_B}{m}\left(\frac{m}{k_B}\frac{p}{\rho}\right) = \frac{1}{\gamma - 1}\frac{p}{\rho}$$

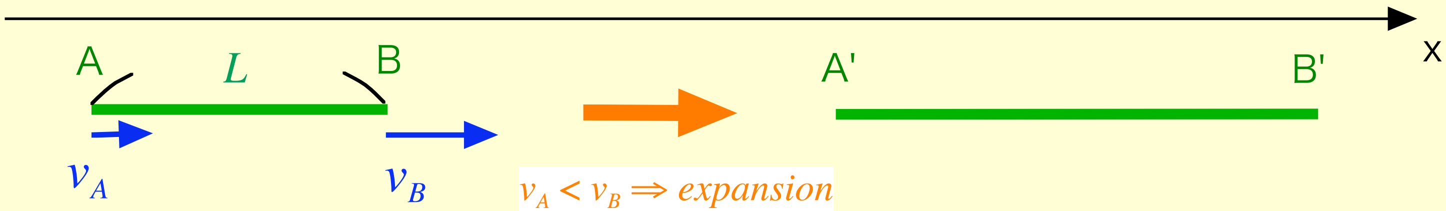
$p = \rho \frac{k_B}{m} T$... equation of state (perfect gas)

$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{\gamma - 1}\frac{p}{\rho}\right)$ ... per unit mass ($\rho V = 1$)

$\frac{1}{V}\frac{dU}{dt} = \rho \frac{d}{dt}\left(\frac{1}{\gamma - 1}\frac{p}{\rho}\right)$ ... per unit volume

$$\frac{1}{V}\frac{dV}{dt}$$ ... **expansion rate**

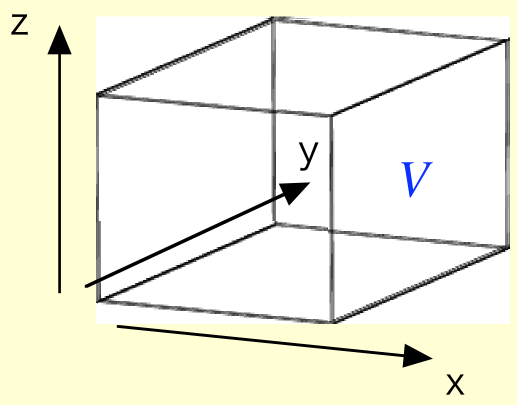
1-dimensional case



$v_A < v_B \Rightarrow expansion$

$$\text{Expansion rate} \equiv \frac{1}{L}\frac{dL}{dt} \Rightarrow \frac{1}{x_B - x_A}\frac{d}{dt}(x_B - x_A) \underset{x_B = x_A + \Delta x}{\Rightarrow} \frac{1}{\Delta x}[v_x(x_A + \Delta x) - v_x(x_A)] \underset{\Delta x \to 0}{\rightarrow} \frac{\partial v_x}{\partial x}$$

3-dimensional case

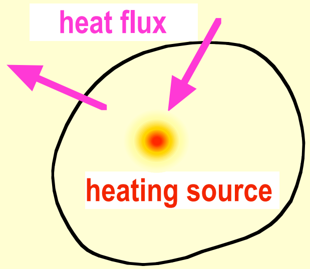


$$\text{Expansion rate} \equiv \frac{1}{V}\frac{dV}{dt} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \equiv \nabla \bullet \boldsymbol{v}$$

$\frac{1}{V} T \frac{dS}{dt}$ ... heating rate

heating = **net heat flux** (external origin) + **heating source** (internal origin)

"net flux" => $-\nabla \bullet$



$$\frac{1}{V} T \frac{dS}{dt} = -\nabla \bullet \boldsymbol{F}_c + \eta j^2 + H$$

per unit volume

thermal conduction flux

$\boldsymbol{F}_c = -\kappa_c \nabla T$

κ_c: *thermal conductivity*

Joule heating source
(from Ohm's law)

Other heating sources
viscous heating
...

$\frac{1}{V} T \frac{dS}{dt} = 0$... **Adiabatic process**

There is **neither** **net heat flux** **nor** **heating source**. **Entropy** $dS = \frac{\delta Q}{T}$ **is conserved**.