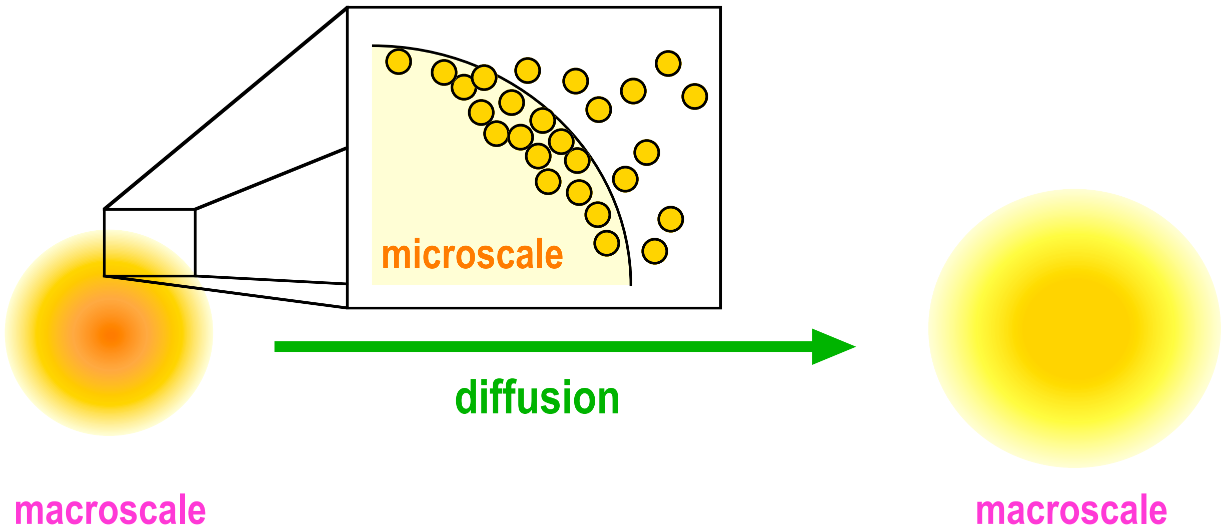
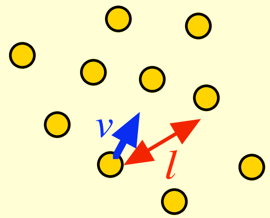
What is diffusion?

A **microscale process (<= kinetic approach)** caused by **collision of particles**, which makes a **macroscale distribution (<= fluid approach) smooth**.



Diffusion is characterized by **particle's motion**:

mean free path l & velocity v



dimension of diffusivity: m^2/s... velocity (m/s) x length (m)

Electrical conductivity & magnetic diffusivity

Electrical conductivity

$$\sigma = 1.53\times 10^{-2}\,\frac{T^{3/2}}{\ln N_D}\ (\text{mho/m}) \propto T^{3/2}$$

conductivity is high in hot plasmas

$\ln N_D$ (Coulomb logarithm) ~ 10

Magnetic diffusivity

$$\eta_{diff} \equiv \frac{1}{\mu_0\sigma} = 5.2\times 10^{7}\,\frac{\ln N_D}{T^{3/2}}\ (\text{m}^2/\text{s}) \propto T^{-3/2}$$

dimension: velocity (m/s) x length (m)

diffusivity is low in hot plasmas

※ What is diffused is magnetic field (or non-gyrating proton).

Electric energy density vs. Magnetic energy density

Electric energy density... $\mathcal{E}_E = \frac{\varepsilon_0}{2} E^2$

Magnetic energy density... $\mathcal{E}_M = \frac{B^2}{2\mu_0}$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \rightarrow \frac{E_0}{l_0} \sim \frac{B_0}{t_0}$$

The ratio is

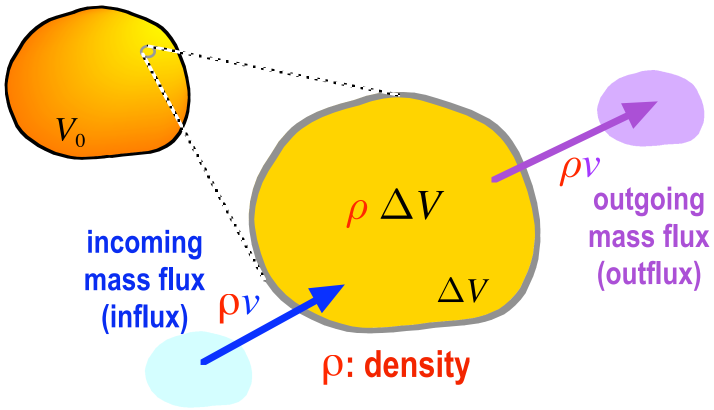
$$\mathcal{E}_E / \mathcal{E}_M = \frac{\varepsilon_0 \mu_0 E^2}{B^2} \approx \frac{\left(\frac{l_0}{t_0} B_0\right)^2}{c^2 B_0^2} = \frac{v_0^2}{c^2} << 1$$

→ **electric energy density** is **negligible when** $v_0 << c$.

- Fluid part
  - mass conservation
  - momentum equation
  - energy equation
  - equation of state
  - Ohm's law
  - charge conservation

(**Field** => **Plasma**)

Equation I... mass conservation



$$\frac{\Delta}{\Delta t}\left(\rho\,\Delta V\right) \Leftarrow \textit{net flux} \equiv \textit{influx} - \textit{outflux}$$

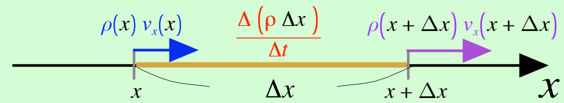
1D case

$$\frac{\Delta\left(\rho\,\Delta x\right)}{\Delta t} = \rho(x)\,v_x(x) - \rho(x+\Delta x)\,v_x(x+\Delta x)$$

$$\Rightarrow \frac{\Delta\,\rho}{\Delta t} = \frac{\rho(x)\,v_x(x) - \rho(x+\Delta x)\,v_x(x+\Delta x)}{\Delta x}$$

Δx, Δt... *independent*

$$\Rightarrow \frac{\partial\rho}{\partial t} = -\frac{\partial}{\partial x}\left(\rho\,v_x\right)$$



3D case

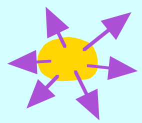
$$\frac{\partial\rho}{\partial t} = -\nabla\bullet\left(\rho\,\boldsymbol{v}\right) \Rightarrow \frac{\partial\rho}{\partial t} + \nabla\bullet\left(\rho\,\boldsymbol{v}\right) = 0$$

$\nabla\bullet\left(\rho\,\boldsymbol{v}\right)$... **– net flux**; when this is positive, gas density decreases with time.

$$\nabla\bullet\left(\rho\,\boldsymbol{v}\right) = \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\bullet\left(\rho\,v_x,\rho\,v_y,\rho\,v_z\right) = \frac{\partial\left(\rho\,v_x\right)}{\partial x} + \frac{\partial\left(\rho\,v_y\right)}{\partial y} + \frac{\partial\left(\rho\,v_z\right)}{\partial z}$$

$$\nabla\bullet\left(\rho\,\boldsymbol{v}\right) > 0 \Rightarrow \frac{\partial\rho}{\partial t} < 0$$

divergence



Mass conservation equation in MHD

$$\frac{\partial \rho^p}{\partial t} + \nabla \bullet \left(\rho^p \boldsymbol{v}^p\right) = 0$$ ... proton's mass conservation $\rho^p = n\,M$

$$\frac{\partial \rho^e}{\partial t} + \nabla \bullet \left(\rho^e \boldsymbol{v}^e\right) = 0$$ ... electron's mass conservation $\rho^e = n\,m$

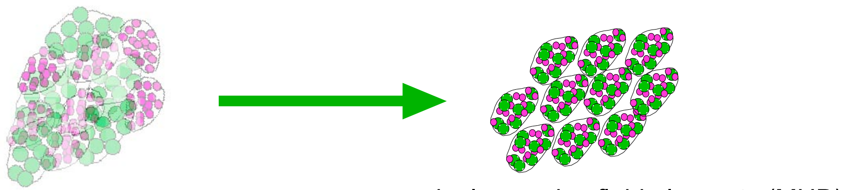
summation

$$\frac{\partial \rho^{MHD}}{\partial t} + \nabla \bullet \left(\rho^{MHD} \boldsymbol{v}^{MHD}\right) = 0$$

Mass conservation eq. in MHD

$\rho^{MHD} \equiv n\left(M + m\right)$: *total density*

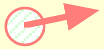
$\boldsymbol{v}^{MHD} \equiv \dfrac{M\,\boldsymbol{v}^p + m\,\boldsymbol{v}^e}{M + m}$: *mass–average velocity*



two-species fluid elements (proton + electron) single-species fluid elements (MHD)

Equation II... momentum equation

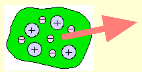
Equation of motion for a particle: $M \frac{d\boldsymbol{v}}{dt} = \boldsymbol{F}$ **(mass** x **acceleration = force)**



$\boldsymbol{F}_{em} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$

$\boldsymbol{F}_g = M\,\boldsymbol{g}$

Momentum equation for a fluid element: $\rho \frac{d\boldsymbol{v}}{dt} = \boldsymbol{f}$



- ρ: **mass density** ($n\,M$; mass per unit volume)
- $\frac{d\boldsymbol{v}}{dt}$: **acceleration**
- $\boldsymbol{f}$: **volume force** (force per unit volume)

- acceleration: $\frac{d\boldsymbol{v}}{dt}$ ... *Lagrangian derivative* $\Rightarrow \frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \bullet \nabla\,\boldsymbol{v}$ ... *Eulerian derivative*
- volume force

  gas pressure gradient force (*collision-related*): $\boldsymbol{f}_p = -\nabla\,p$

  Coulomb force + Lorentz force (*collision-unrelated; interaction with field*): $\boldsymbol{f}_{em} = n\,q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right)$

  gravitational force (*collision-unrelated; interaction with field*): $\boldsymbol{f}_g = n\,M\,\boldsymbol{g}$

  viscous force (*collision-related*): $\boldsymbol{f}_v = \rho\,\nu\left[\nabla^2\,\boldsymbol{v} + \frac{1}{3}\nabla\left(\nabla \bullet \boldsymbol{v}\right)\right]$, ν: *viscosity*