$$F_z \approx -\frac{m v_\theta^2}{2 B(0,z)} \frac{\partial B(0,z)}{\partial z}$$

$= \mu$ (magnetic moment)

general form

$$F_{/\!/} \approx -\mu \nabla_{/\!/} B$$

$$\mu = \frac{m v_\perp^2 / 2}{B}$$

※ Check the slide "**Magnetic moment**"

$$F = -\nabla U = -\nabla(-\mu \bullet B) = -|\mu| \nabla B$$

$$\mu = -|\mu| \frac{B}{B} \quad |\mu| \sim const.$$

gyrating charged particle => diamagnetic magnetic moment

Equation of motion in $B_{/\!/}-direction$: $m \frac{d v_{/\!/}}{dt} = F_{/\!/} \approx -\mu \nabla_{/\!/} B$

take an inner product with $v_{/\!/}$

$$v_{/\!/} \bullet \nabla_{/\!/} B = \frac{dB}{dt} - \frac{\partial B}{\partial t} = \frac{dB}{dt}$$

$\frac{\partial B}{\partial t} = 0$

Static field (B does not change with time)

$$\frac{d}{dt}\left(\frac{1}{2} m v_{/\!/}^2\right) \sim -\mu \frac{dB}{dt}$$

$$\frac{d}{dt}(\mu B) = \frac{d}{dt}\left(\frac{m v_\perp^2}{2B} B\right) = \frac{d}{dt}\left(\frac{m v_\perp^2}{2}\right) = -\frac{d}{dt}\left(\frac{1}{2} m v_{/\!/}^2\right)$$

Conservation of total kinetic energy: $\frac{d}{dt}\left(\frac{1}{2} m v_{/\!/}^2 + \frac{1}{2} m v_\perp^2\right) = 0$

Lorentz force does not do any work on a particle.
$v_{/\!/}^2 + v_\perp^2 = v^2 = const.$

$$\frac{d\mu}{dt} \sim 0$$

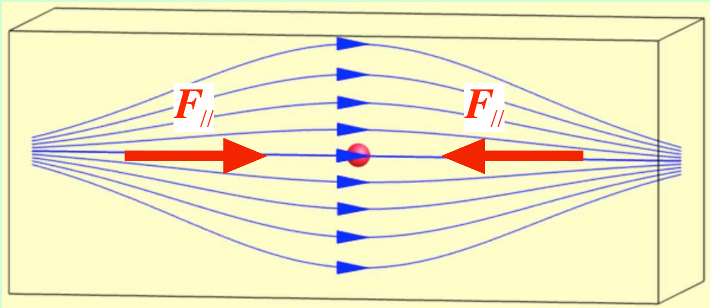
Conservation of magnetic moment

When the spatial variation of **B** felt by a charged particle while taking one gyration is sufficiently small,

its **magnetic moment** μ becomes an **adiabatic invariant**: $\frac{d\mu}{dt} \sim 0$ → $\mu = \frac{m v_\perp^2 / 2}{B} = const.$

$$\frac{d}{dt}\left(\frac{1}{2} m v_{//}^2\right) \sim -\mu \frac{dB}{dt}$$ → when B increases, $v_{//}$ decreases.

μ and m are constant



$v_{//}^2 + v_\perp^2 = v^2 = const.$

← B increases, $v_{//}$ decreases, $v_\perp$ increases

→ B increases, $v_{//}$ decreases, $v_\perp$ increases

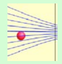
When $v_{//}$ becomes 0, $F_{//} \approx -\mu \nabla_{//} B$ drives the reflection of the particle.

Mirror effect

ϕ_c: **critical angle**

$v_{//} = 0$ (*reflection point*) with $B = B_{max}$

$\phi = \frac{\pi}{2}$



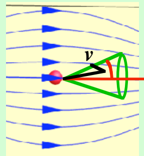
$$\frac{m (v \sin \phi_c)^2}{2 B_{min}} = \frac{m \left(v \sin\left(\frac{\pi}{2}\right)\right)^2}{2 B_{max}} = const.$$

central point with $B = B_{min}$

$$\sin^2 \phi_c = \frac{B_{min}}{B_{max}}$$

$\phi > \phi_c \Rightarrow$ *reflect*

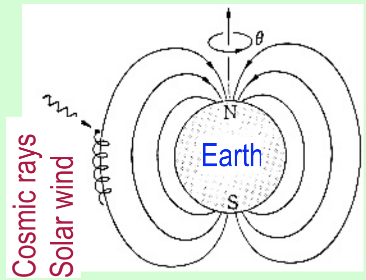
$\phi < \phi_c \Rightarrow$ *escape*



loss cone

$\phi < \phi_c$

When $v_{//}$ is preferentially accelerated, the particle may enter the loss cone.



Example: magnetosphere

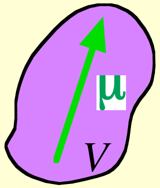
Magnetic moment

angular momentum of a charged particle

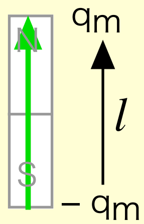
Magnetic (dipole) moment...

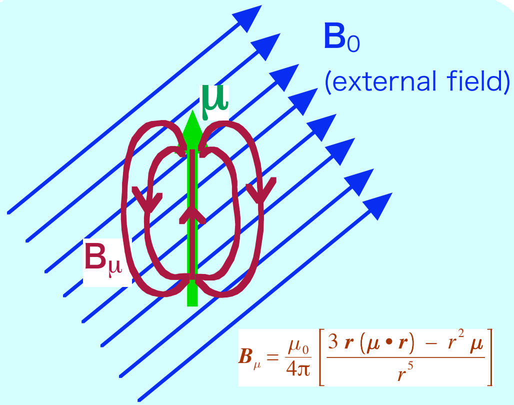
Definition of **magnetic moment**:

$$\mu \equiv \frac{1}{2}\int_V \boldsymbol{r} \times \boldsymbol{j}\, d\boldsymbol{r}^3$$



$$\boldsymbol{\mu} = q_m\, \boldsymbol{l}$$





$$\boldsymbol{B}_\mu = \frac{\mu_0}{4\pi}\left[\frac{3\,\boldsymbol{r}\,(\boldsymbol{\mu}\bullet\boldsymbol{r}) - r^2\,\boldsymbol{\mu}}{r^5}\right]$$

potential energy: $U = -\,\boldsymbol{\mu}\cdot\boldsymbol{B}_0$

force: $\boldsymbol{F} = -\,\nabla\, U$

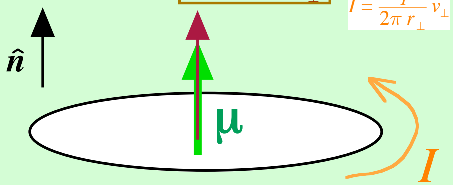
torque: $\boldsymbol{N} = \boldsymbol{\mu} \times \boldsymbol{B}_0$

Magnetic moment *is related to* **angular momentum**:

Circular current

$$B_\mu^{center} = \frac{\mu_0}{2}\frac{I}{r_\perp}$$

$$I = \frac{q}{2\pi\, r_\perp} v_\perp$$

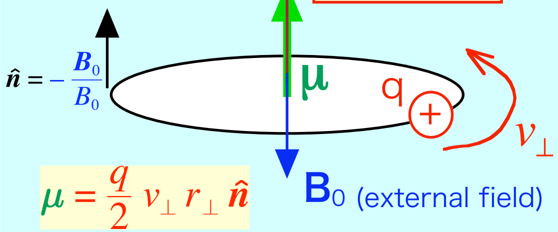


$$\mu = I\, S\, \hat{\boldsymbol{n}} = I\,\pi\, r_\perp^2\, \hat{\boldsymbol{n}}$$

Gyrating charged particle

$$B_\mu^{center} = \frac{\mu_0\, q}{4\pi\, r_\perp^2} v_\perp$$

diamagnetic



$$\mu = \frac{q}{2} v_\perp\, r_\perp\, \hat{\boldsymbol{n}}$$

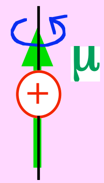
$$= \frac{q}{2m}\boldsymbol{L} \qquad v_\perp r_\perp \hat{\boldsymbol{n}} = \boldsymbol{L}/m$$

$$= I\,\pi\, r_\perp^2\, \hat{\boldsymbol{n}} \qquad I = \frac{q}{2\pi\, r_\perp} v_\perp$$

$$= -\frac{m\, v_\perp^2/2}{B_0}\frac{\boldsymbol{B}_0}{B_0} \qquad r_\perp = \frac{v_\perp}{\omega_B}$$

$$= -\frac{1}{2\pi}\frac{q^2}{m}\Phi_B \frac{\boldsymbol{B}_0}{B_0} \qquad \Phi_B = \pi r_\perp^2\, B_0$$

Spinning charged particle



$$\mu = \frac{q}{2\,m}\boldsymbol{L}_S$$

angular momentum $\boldsymbol{L}_S$

... quantized by $\hbar$