Motion of a charged particle in $B_{//}$– *direction*

(Mirror effect)

Mirror effect

$\boldsymbol{B}$: magnetic field (nonuniform & constant, weakly curved shape)
$\boldsymbol{v}$: particle's velocity
m, q: particle's mass & charge

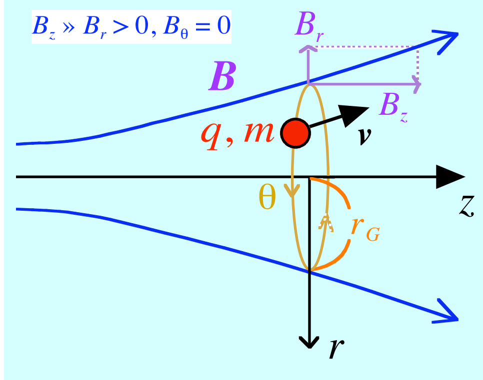
Gyration-average Lorentz force $\langle q\, \boldsymbol{v} \times \boldsymbol{B} \rangle$:

$$\boldsymbol{F} = q\left(v_r\, \hat{r} + v_\theta\, \hat{\theta} + v_z\, \hat{z}\right) \times \left(B_r\, \hat{r} + B_z\, \hat{z}\right)$$
$$= q\left(v_r B_z\, \hat{r} \times \hat{z} + v_\theta B_r\, \hat{\theta} \times \hat{r} + v_\theta B_z\, \hat{\theta} \times \hat{z} + v_z B_r\, \hat{z} \times \hat{r}\right)$$
$$= q\left[v_\theta B_z\, \hat{r} + \left(v_z B_r - v_r B_z\right) \hat{\theta} - v_\theta B_r\, \hat{z}\right]$$

$\langle \hat{r} \rangle = 0, \langle \hat{\theta} \rangle = 0$ $\quad q \gtrless 0 \rightarrow v_\theta \lessgtr 0$

$$\langle \boldsymbol{F} \rangle_{gyration} = |q|\,|v_\theta|\, B_r\, \hat{z}$$

$$F_z \approx |q|\,|v_\theta|\, B_r\left(r_G, z\right)$$

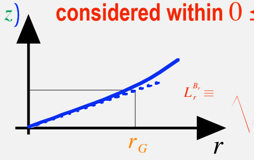


Taylor expansion of $B_r(r, z)$ around $r = 0$:

odd function of r

$$B_r(r, z) = \underbrace{B_r(0, z)}_{=0} + \left[\frac{\partial B_r(r, z)}{\partial r}\right]_{r \to 0} r + \underbrace{\ldots}_{\sim 0}$$

0th and/or 1st-order terms are considered within $0 \le r \le r_G$.



$$L_r^{B_r} \equiv \sqrt{\frac{\left|\left(\frac{\partial B_r}{\partial r}\right)_{r \to 0}\right|}{\left|\left(\frac{\partial^3 B_r}{\partial r^3}\right)_{r \to 0}\right|}} \gg r_G$$

$$\left[\frac{\partial B_r}{\partial r}\right]_{r \to 0} \approx \left[\frac{\partial B_r}{\partial r}\right]_{r = r_G}$$

$r = r_G$

$$\frac{B_r(r_G, z)}{r_G} \approx \left[\frac{\partial B_r(r, z)}{\partial r}\right]$$

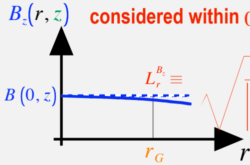
Taylor expansion of $B_z(r, z)$ around $r = 0$:

even function of r

$$B_z(r, z) = \underbrace{B_z(0, z)}_{= B(0, z)} + \underbrace{\left[\frac{\partial B_z(r, z)}{\partial r}\right]_{r \to 0}}_{=0} r + \ldots$$

1st-order term is 0 when $r \to 0$ (for smoothness)

0th and/or 1st-order terms are considered within $0 \le r \le r_B$.



$$L_r^{B_z} \equiv \sqrt{\frac{\left|B_z(r = 0)\right|}{\left|\left(\frac{\partial^2 B_z}{\partial r^2}\right)_{r \to 0}\right|}} \gg r_G$$

Divergence free condition ($\nabla \bullet \boldsymbol{B} = 0$) at $r = r_G$:

$$\nabla \bullet \boldsymbol{B}\Big|_{r = r_G} = \left[\frac{\partial B_r(r, z)}{\partial r}\right]_{r = r_G} + \frac{B_r(r_G, z)}{r_G} + \left[\frac{\partial B_z(r, z)}{\partial z}\right]_{r = r_G} = 0$$

$$\frac{B_r(r_G, z)}{r_G} \approx -\frac{1}{2}\left[\frac{\partial B_z(r, z)}{\partial z}\right]_{r = r_G} \approx -\frac{1}{2}\frac{\partial B(0, z)}{\partial z}$$

$$\nabla \bullet \diamond = \frac{\partial}{\partial r}\left(\diamond_r\right) + \frac{\diamond_r}{r} + \frac{1}{r}\frac{\partial \diamond_\theta}{\partial \theta} + \frac{\partial \diamond_z}{\partial z}$$

Formula in cylindrical coordinates

$$B_r\left(r_G, z\right) \approx -\frac{r_G}{2}\frac{\partial B(0, z)}{\partial z}$$

$$F_z \approx -\frac{m\, v_\theta^2}{2\, B(0, z)}\frac{\partial B(0, z)}{\partial z}$$

$$r_G = \frac{|v_\theta|}{\omega_B}, \quad \omega_B \equiv \frac{|q|\, B(0, z)}{m}$$