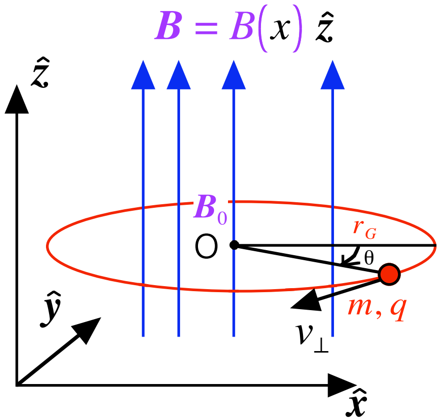
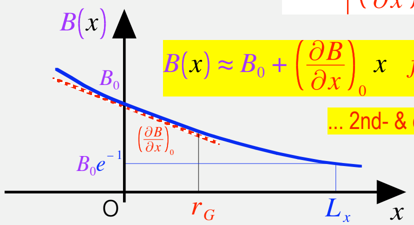
Gradient-B drift



$\boldsymbol{B}$: magnetic field (nonuniform (weak variation) & constant, straight shape)

$\boldsymbol{v}$: particle's velocity m, q: particle's mass & charge

$\boldsymbol{B} = B(x)\,\hat{\boldsymbol{z}}$ => weak variation: $L_x \equiv \dfrac{B_0}{\left|\left(\dfrac{\partial B}{\partial x}\right)_0\right|} \gg r_G$ $\left(\dfrac{\left|\left(\dfrac{\partial B}{\partial x}\right)_0 r_G\right|}{B_0} \ll 1 \;\Rightarrow\; r_G \ll L_x\right)$



$B(x) \approx B_0 + \left(\dfrac{\partial B}{\partial x}\right)_0 x \quad for\ |x| \le r_G \ll L_x$

... 2nd- & even higher-order terms are neglected.

Equation of motion in $B_\perp$–*plane*: $m\dfrac{d\boldsymbol{v}_\perp}{dt} = q\,\boldsymbol{v}_\perp \times \boldsymbol{B}$

Gyration-average: (remove gyration effect)

$\langle f(\theta)\rangle \equiv \dfrac{\int_0^{2\pi} f(\theta)\,d\theta}{\int_0^{2\pi} d\theta}$

=> only x-component of force is taken into account

$\langle \cos^2\theta\rangle = \dfrac{\int_0^{2\pi}\cos^2\theta\,d\theta}{\int_0^{2\pi} d\theta} = \dfrac{1}{2}$

$\boldsymbol{B}(x) \approx \boldsymbol{B}_0 + \left(\dfrac{\partial B}{\partial x}\right)_0 x\,\hat{\boldsymbol{z}}$

$q\,\boldsymbol{v}_\perp \times \boldsymbol{B}$ → $q\,\boldsymbol{v}_\perp \times \boldsymbol{B}_0 + q\,\boldsymbol{v}_\perp \times \left[\left(\dfrac{\partial B}{\partial x}\right)_0 x\,\hat{\boldsymbol{z}}\right]$

$\boldsymbol{v}_G(t) \approx -v_\perp \sin\theta\,\hat{\boldsymbol{x}} - v_\perp\cos\theta\,\hat{\boldsymbol{y}}$

$\left(\dfrac{\partial B}{\partial x}\right)_0 r_G \cos\theta\,\hat{\boldsymbol{z}}$

$\boldsymbol{v}_\perp = \boldsymbol{v}_G(t) + \Delta\boldsymbol{v}$ → $\Delta v\left(\dfrac{\partial B}{\partial x}\right)_0 x = \Delta v\,\Delta B \sim 0$

assume $|\boldsymbol{v}_G(t)| \gg |\Delta\boldsymbol{v}|$

$r_G = \dfrac{v_\perp}{\omega_B} = \dfrac{v_\perp m}{q B_0}$ $\left(\dfrac{\partial B}{\partial x}\right)_0 \hat{\boldsymbol{x}} = \nabla_0 B$

$\boldsymbol{F}_{\nabla B} = -\dfrac{m v_\perp^2}{2 B_0}\nabla_0 B$: **gradient-B force**

$\boldsymbol{F}_{\nabla B} = -\mu\,\nabla_0 B$ $\mu = \dfrac{m v_\perp^2/2}{B_0}$

when $\boldsymbol{F}_{\nabla B}$ is uniform & constant => **gradient-B drift**

$$\boldsymbol{v}_{\nabla B} = \frac{\boldsymbol{F}_{\nabla B} \times \boldsymbol{B}_0}{q B_0^2} = -\frac{m v_\perp^2}{2 q B_0^3}(\nabla_0 B) \times \boldsymbol{B}_0$$

$$\boldsymbol{v}_\perp = \boldsymbol{v}_G(t)\ \textbf{(gyration)} + \boldsymbol{v}_{\nabla B}\ \textbf{(grad-B drift)}$$

validation ← $\dfrac{v_{\nabla B}}{v_G} \sim \dfrac{v_G}{\omega_B}\dfrac{1}{L_x} = \dfrac{r_G}{L_x} \ll 1$