Characteristic scales related to magnetic field

(gyration)

In plasmas, charge density is always close to zero, but electric current density is not.

$$\rho_c = e\left(Z_i n_i - n_e\right) \sim 0$$

$$E_{coulomb}(r) = \iiint_{V_0} \frac{\rho_C(r')(r - r')}{|r - r'|^3} dV'$$ CGS unit

Coulombic electric field does not exist globally.

$$j = Z_i n_i e v_i + n_e(-e) v_e \sim e n_e (v_i - v_e) \neq 0$$

$$B(r) = \frac{1}{c^2}\iiint_{V_0} \frac{j(r') \times (r - r')}{|r - r'|^3} dV'$$ CGS unit

Magnetic field can globally exist.

Cyclotron (gyro-) frequency... charged particle gyrates around a magnetic field line

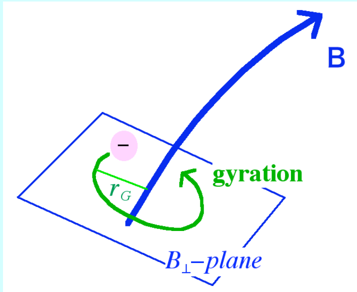
$$m \frac{d v_\perp}{dt} = \frac{q}{c} v_\perp \times B \Rightarrow m r_G \omega_B^2 = \frac{q}{c} r_G \omega_B B$$

$$v_\perp = r_G \omega_B$$

$$\omega_B \equiv \frac{q B}{m c}$$

$$\omega_B \equiv \frac{q B}{m} \ (MKS\ unit)$$

$$\nu_B \equiv \frac{\omega_B}{2\pi}$$



Magnetic field also introduces characteristic time & length scales into plasmas...

Time scale... **gyration time:** $t_G = \frac{2\pi}{\omega_B} \sim \frac{r_G}{v_T}$ ($v_\perp \sim v_T$)

When $t_G << t_c$, a particle is **strongly coupled** with magnetic field

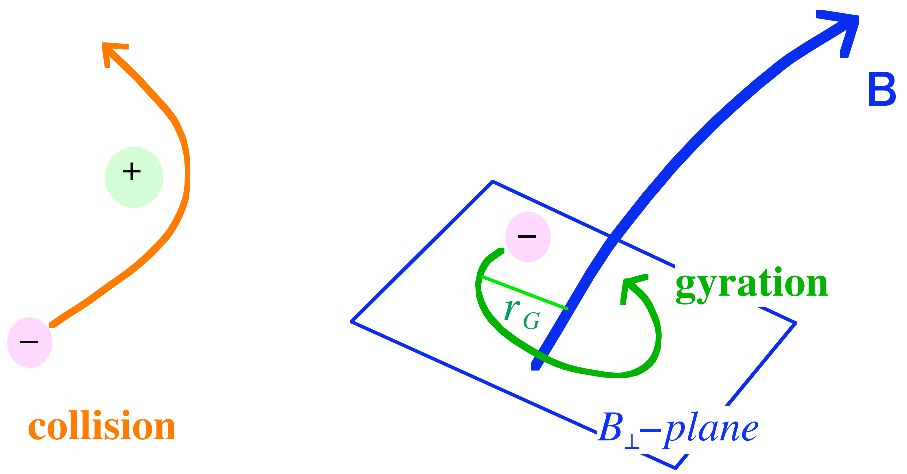
Length scale... **gyration radius:** $r_G = \frac{v_\perp}{\omega_B} \sim \frac{v_T}{\omega_B}$

When $r_G << l_{mfp}$, magnetic field plays a main role in **changing the momentum of a particle in $B_\perp$-plane.**

Collision vs. Gyration

macroscale quantity · microscale quantity

field-line curvature radius $\gg r_G$
is assumed



Gyration plays a **collision-like role** in changing the momentum of a charged particle in $B_\perp$−plane.

Difference between collision & gyration:

collision produces **random motion** in **all directions** => **pressure**, **temperature in all directions**
gyration produces **ordered motion** in $B_\perp$−plane => **pressure**, **electric current in** $B_\perp$−plane

Summary of characteristic scales of plasmas

	E-related scale			**B**-related scale	
Length	**mean free path** (l_{mfp})	**Debye length** (L_D)	**effective radius** (r_{eff})	**gyration radius** (r_G)	**mean interval** (l)
Time	**collision time** ($t_c \sim \frac{l_{mfp}}{v_T}$)	**oscillation period** ($t_p \sim \frac{L_D}{v_T}$)		**gyration period** ($t_G \sim \frac{r_G}{v_T}$)	
Physical process	**collision**	**oscillation**	**collision with a large scattering angle**	**gyration**	
Physical effect	**thermalization**	**neutralization**	**thermalization**	**pressure & current in $B_\perp$-plane**	
CGS unit	$l_{mfp}^{e-e} \sim \frac{m_e^2 v_{Te}^4}{n_e e^4} \frac{1}{\ln \Lambda}$	$L_D \sim \sqrt{\frac{k_B T}{4\pi e^2 n_e}}$	$r_{eff} \sim \frac{e^2}{k_B T}$	$r_G \sim \frac{m c v_T}{e B}$	$l \sim n^{-\frac{1}{3}}$

Magnitude relationship between characteristic scales

Length scales:

$l_{mfp}^{e-e} \sim l_{mfp}^{e-i} \sim l_{mfp}^{i-i} \ll l_{mfp}^{i-e}$ ($\times \sqrt{\frac{M}{m}}$)

$$r_{eff} \ll \bar{l} \ll L_D \ll l_{mfp}^{e-e} \quad \longleftrightarrow \quad r_G \sim \frac{v_{Te}}{\nu_B}$$

This is the length scale related to **B**, so it is independent from the length scales related to **E**.

$$\frac{e^2}{n_e^{-1/3}} \ll k_B T_e \sim \frac{e^2}{r_{eff}}$$

$$\bar{l} \sim n_e^{-\frac{1}{3}}$$

$$N_D \sim n_e L_D^3 = \left(\frac{L_D}{\bar{l}}\right)^3 \gg 1$$

$$l_{mfp}^{e-e} \approx \frac{m_e^2 v_e^4}{n_e e^4} \frac{1}{\ln \Lambda} \sim \frac{1}{n_e r_{eff}^2} \frac{1}{\ln N_D} \sim \frac{N_D^2}{n_e L_D^2} \frac{1}{\ln N_D} \sim \frac{N_D}{\ln N_D} L_D \gg L_D$$

$\Lambda \sim N_D$

$\frac{L_D}{r_{eff}} \sim N_D$

$n_e L_D^2 \sim \frac{N_D}{L_D}$

Time scales:

$$t_c^{e-e} \sim \frac{l_{mfp}^{e-e}}{v_{Te}}$$

$$t_p \sim \frac{L_D}{v_{Te}}$$

$$\Rightarrow \frac{t_c^{e-e}}{t_p} \sim \frac{l_{mfp}^{e-e}}{L_D} \sim \frac{N_D}{\ln N_D} \gg 1$$

$$t_p \ll t_c \quad \longleftrightarrow \quad t_G \equiv 2\pi \frac{m\,c}{q\,B} \sim \frac{r_G}{v_{Te}}$$

This is the time scale related to **B**, so it is independent from the time scales related to **E**.

Plasma relaxes to a charge neutral state **much faster than** a thermal state.

=> nonthermal plasma ($t_p \ll t_{phenomenon} \ll t_c$), **thermal plasma** ($t_p \ll t_c \ll t_{phenomenon}$)