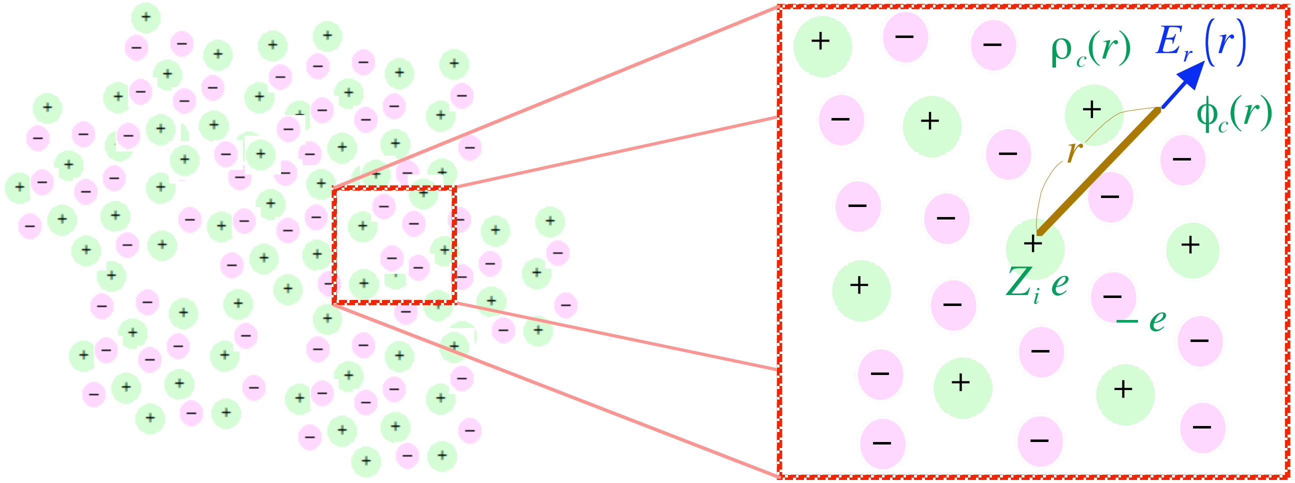
# *Debye length*

## Isotropic distribution model

... only depends on the distance $r$ from the central particle

(each particle continuously moves, while keeping the same isotropic distribution)




*$Z_i$... ion's valency number*

($Z_i = 1$ for a proton)

$\phi_c$: *electrostatic potential*, $\rho_c$: *charge density*... both depend on $r$

CGS unit

Equation for electrostatic potential... $\nabla^2\phi_c = -4\pi\rho_c(r)$

Charge density distribution (thermal state) => $\rho_c(r) = Z_i\,e\,\delta(r) + (-e)\,n_e\,e^{-\frac{(-e)\phi_c}{k_B T_e}} + (Z_i\,e)\,n_i\,e^{-\frac{(Z_i e)\phi_c}{k_B T_i}}$

central ion

surrounding particles

*$n_e, n_i$... number density of electron and ion when $\phi_c \to 0$ ($r \to \infty$)*

Assumption: electrostatic energy « thermal energy

$|-e\,\phi_c| \ll k_B T_e,\ |Z_i\,e\,\phi_c| \ll k_B T_i$

$e^x \approx 1 + x$ *when* $|x| \ll 1$

$Z_i\,n_i = n_e$

$T \equiv \frac{T_e (Z_i T_i)}{T_e + (Z_i T_i)}$ reduced temperature

$-\frac{1}{4\pi L_D^2}\phi_c$

## Debye length

$$L_D \equiv \sqrt{\frac{k_B T}{4\pi e^2 n_e}}$$

*(MKS unit)*

$$L_D \equiv \sqrt{\frac{\varepsilon_0 k_B T}{e^2 n_e}}$$

# *Debye shielding*

$$\nabla^2 \phi_c = -4\pi\rho_c(r)$$

isotropic distribution $\frac{\partial}{\partial\theta}=0, \frac{\partial}{\partial\phi}=0$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi_c}{dr}\right) = -4\pi Z_i e\,\delta(\boldsymbol{r}) + \frac{\phi_c}{L_D^2}$$

spherical coordinate system

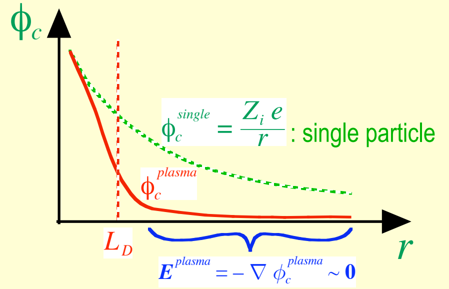
$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

Transform the function: $\Phi = r\,\phi_c$ => $\frac{d^2\Phi}{dr^2} = \frac{\Phi}{L_D^2}\ (0 < r < \infty)$

Boundary condition:

$\phi_c \to \frac{Z_i e}{r}$ *when* $r \to 0$, $\phi_c \to 0$ *when* $r \to \infty$

$$\phi_c^{plasma} = \frac{Z_i e}{r}\, e^{-\frac{r}{L_D}}$$




$\rho_c^{plasma}(r) \sim Z_i e\,\delta(r) - \frac{1}{4\pi L_D^2}\phi_c = Z_i e\,\delta(r) - \frac{Z_i e}{4\pi L_D^2}\frac{1}{r}e^{-\frac{r}{L_D}}$ => when $r \gg L_D$, $\rho_c^{plasma}(r) \sim 0$ (**local charge neutrality**)

Surrounding ions and electrons **shield Coulombic electric field generated by each ion and electron**, which significantly reduces the range of Coulomb force in plasma systems.

**=> inhibit the existence of global Coulombic electric field in plasma systems**

$$L_D \equiv \sqrt{\frac{k_B T}{4\pi e^2 n_e}} \qquad T \equiv \frac{T_e (Z_i T_i)}{T_e + (Z_i T_i)}$$

Ion with a large valency number weakly shields electric field by increasing $L_D$ (T becomes large).