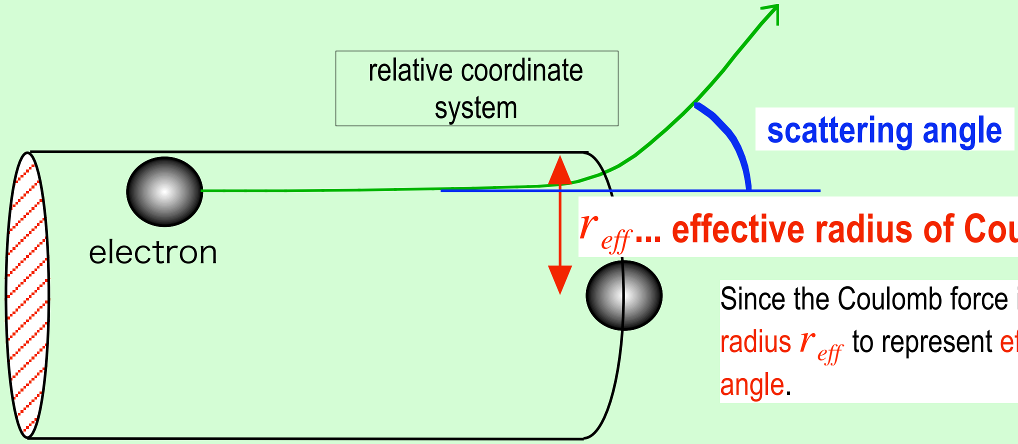
Mean free path of a charged particle (collision without a direct touch)

General definition of collision... **a process causing a change in momentum (or velocity)**



r_{eff}**... effective radius of Coulombic collision**

Since the Coulomb force is long-range force, we define effective radius r_{eff} to represent effective collision with a large scattering angle.

How to estimate r_{eff}... **particle's kinetic energy (~ $m_e v_e^2$) ~ particle's Coulombic energy ($\frac{e^2}{r_{eff}}$)**

=> scattering angle ~ 90º

$$\frac{e^2}{r_{eff}} \sim m_e v_e^2 \Rightarrow r_{eff} \sim \frac{e^2}{m_e v_e^2}$$

thermal plasma $m_e v_e^2 \sim k_B T_e$ →

$$r_{eff} \sim \frac{e^2}{k_B T_e} \sim 1.6\times10^{-11}\ \mathrm{m} \quad when\ T_e = 10^6\ K$$

$$\gg r_e \sim 10^{-15}\ \mathrm{m}$$

$$\sigma^{e-e} \sim \pi r_{eff}^2 \approx \pi \left(\frac{e^2}{m_e v_e^2}\right)^2$$

$$l_{mfp}^{e-e\,*} \equiv \frac{1}{n_e \sigma^{e-e}} \sim \frac{m_e^2 v_e^4}{n_e e^4}$$

thermal plasma $m_e v_e^2 \sim k_B T_e$ →

$$l_{mfp}^{e-e} \sim \frac{(k_B T_e)^2}{n_e e^4}$$

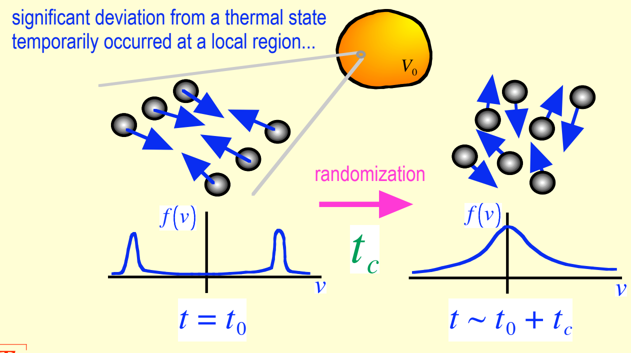
* More precisely (including non-effective collision), $l_{mfp}^{e-e} \sim \frac{m_e^2 v_e^4}{n_e e^4} \frac{1}{\ln \Lambda}$, where $\ln \Lambda \sim 10$ (Coulomb logarithm)

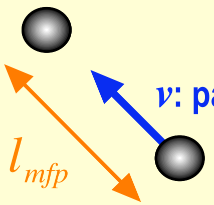
Collision time & collision frequency

Collision time t_c ... **Time required for a particle to collide with another particle (<u>relax into a thermal state via collision</u>)**

(Thermalization time, Thermal relaxation time)

$$t_c = \frac{l_{mfp}}{v}$$





v: particle's (relative) velocity $\sim v_T \equiv \sqrt{\frac{k_B T}{m}}$

(particle's kinetic energy is ~ $k_B T$)

$$t_c^{e-e} = \frac{l_{mfp}^{e-e}}{v_e} \sim \frac{m_e^2 v_e^3}{n_e e^4} = \frac{m_e^{1/2} (k_B T_e)^{3/2}}{n_e e^4} \propto m_e^{1/2} \sim 0.92 \frac{T_e^{3/2}}{n_e} \text{ (s)}$$

Collision frequency ν_c ... **number of collisions per unit time**

$$\nu_c = \frac{1}{t_c}$$