Three components of energy flux... $F_{rad}, F_{cond}, F_{conv}$

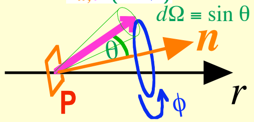
- **Radiative flux** (by **photon**): $F_{rad} = \int_0^\infty F_{\boldsymbol{r},\nu}(r)\, d\nu$

$$F_{\boldsymbol{n},\nu} = \int I_{\boldsymbol{n},\nu}(\theta,\phi) \cos\theta \, d\Omega$$

flux passing through a plane with normal direction $\boldsymbol{n}$

$I_{\boldsymbol{n},\nu}(\theta,\phi)$

$d\Omega \equiv \sin\theta \, d\theta \, d\phi$: solid angle



Black-body radiation field is assumed in the solar interior:

radiative intensity $I_{\boldsymbol{n},\nu}(\theta,\phi)$ is in thermal equilibrium with particles => only depends on **temperature**

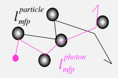
Rosseland approximation

$$\begin{cases} I^{0th}_{\boldsymbol{r},\nu} = \dfrac{2h\nu^3}{c^2} \dfrac{1}{e^{\frac{h\nu}{k_B T}} - 1} = B_\nu(T) \text{... isotropic} \\ I^{1st}_{\boldsymbol{r},\nu} = B_\nu(T) + \cos\theta \left(l^{photon}_{mfp} \dfrac{dB_\nu}{dr} \right) \end{cases}$$

$$B_\nu(T) \equiv \frac{2h}{c^2} \frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1} \text{ : Kirchhoff–Planck function}$$

(collision <=> absorption, reemission, scattering: $l^{photon}_{mfp} \sim \dfrac{1}{\kappa_\nu \rho} \ll L_{sys}$)

κ_ν : *absorption coefficient for photon with* ν



thermal state of particle
<= Maxwellian distribution function $f_{MW}(v) \propto e^{-\frac{mv^2}{2k_B T}}$

thermal state of photon
<= Planck's distribution function $n_{BE}(\nu) = \dfrac{1}{e^{\frac{h\nu}{k_B T}} - 1}$

$$F_{\boldsymbol{r},\nu}(r) = -\frac{4\pi}{3\kappa_\nu \rho} \frac{dB_\nu}{dT} \frac{dT}{dr}$$

flux passing through a plane with normal direction r

$$F_{rad} = \int_0^\infty F_{\boldsymbol{r},\nu}(r)\, d\nu = -\frac{4\pi}{3\kappa_R \rho} \frac{d}{dT}\left(\frac{\sigma T^4}{\pi}\right) \frac{dT}{dr} = -\frac{16\sigma}{3\kappa_R \rho} T^3 \frac{dT}{dr}$$

total (radiative) flux passing through a plane with normal direction r

σ : *Stefan–Boltzmann coefficient*

$$\frac{1}{\kappa_R} \equiv \frac{\displaystyle\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu}{\displaystyle\int_0^\infty \frac{dB_\nu}{dT} d\nu} \text{ : Rosseland mean opacity}$$

average in frequency space

weight function: $\dfrac{dB_\nu}{dT}$

- **Conductive flux** (by **free electron**): $F_{cond} = -\kappa_c \dfrac{dT}{dr} = -\dfrac{16\sigma}{3\kappa_e \rho} T^3 \dfrac{dT}{dr}$

κ_c : *electron conductivity* $\quad$ κ_e : *electron conduction opacity*

$$\kappa_e = \frac{16\sigma T^3}{3\rho\, \kappa_c}$$

$$F_{rad} + F_{cond} = -\frac{16\sigma}{3\kappa\rho} T^3 \frac{dT}{dr}$$

$\kappa \equiv \dfrac{\kappa_R \kappa_e}{\kappa_R + \kappa_e}$: **reduced opacity**

- **Convective flux** (by **parcel**): $F_{conv} = \rho U v_{conv}$... We have to express this **without using velocity** (=> **mixing length theory**).

U: internal energy per unit mass

• **Energy conservation** (spherically symmetric)**...**

$$\nabla \cdot \boldsymbol{F}_{conv} = \rho\varepsilon - \nabla \cdot \left(\boldsymbol{F}_{rad} + \boldsymbol{F}_{cond}\right) + \rho\, T \left(\frac{ds}{dt}\right)_{im}$$

integrate with
θ, ϕ *at* $r = r$

$$\frac{d}{dr}\left[4\,\pi\, r^2 \left(F_{rad} + F_{cond} + F_{conv}\right)\right] = 4\,\pi\, r^2\left[\rho\varepsilon + \rho\, T\left(\frac{ds}{dt}\right)_{im}\right]$$

$$\frac{1}{4\,\pi\, r^2\,\rho}\frac{d}{dr}\left[4\,\pi\, r^2 \left(F_{rad} + F_{cond} + F_{conv}\right)\right] = \varepsilon + T\left(\frac{ds}{dt}\right)_{im}$$

$$L \equiv \oint_S \left(\boldsymbol{F}_{rad} + \boldsymbol{F}_{cond} + \boldsymbol{F}_{conv}\right) \cdot \boldsymbol{dS}: \textit{luminosity}$$

$$L = 4\pi r^2 \left(F_{rad} + F_{cond} + F_{conv}\right)$$

(spherically symmetric case)

$$\frac{dL}{dm} = \varepsilon + T\left(\frac{ds}{dt}\right)_{im}$$

About $\frac{dT}{dm}$...

In the core & radiative zone (F_{rad} and F_{cond} are dominant)...

$$\frac{L}{4\pi r^2} = F_{rad} + F_{cond} = -\frac{16\sigma}{3\kappa\rho} T^3 \frac{dT}{dr}$$

$$\frac{dr}{dm} = \frac{1}{4\pi\rho r^2}$$

$$\frac{dT}{dr} = 4\pi\rho r^2 \frac{dT}{dm}$$

$$\frac{dT}{dm} = -\frac{3\kappa L}{256\pi^2 \sigma r^4 T^3}$$

In the convection zone (F_{conv} is dominant)...

$$\frac{L}{4\pi r^2} = F_{conv} \longrightarrow \frac{dT}{dm} = \left(\frac{dT}{dm}\right)_{conv}$$

close to adiabatic temperature gradient: $\left(\frac{dT}{dm}\right)_{conv} \sim \left(\frac{dT}{dm}\right)_{ad}$

※ Derivation of the temperature gradient $\left(\frac{dT}{dm}\right)_{conv}$ using mixing length theory will be explained in the section of the convection zone (express the convective flux in a diffusion form, like the radiative and conductive flux).