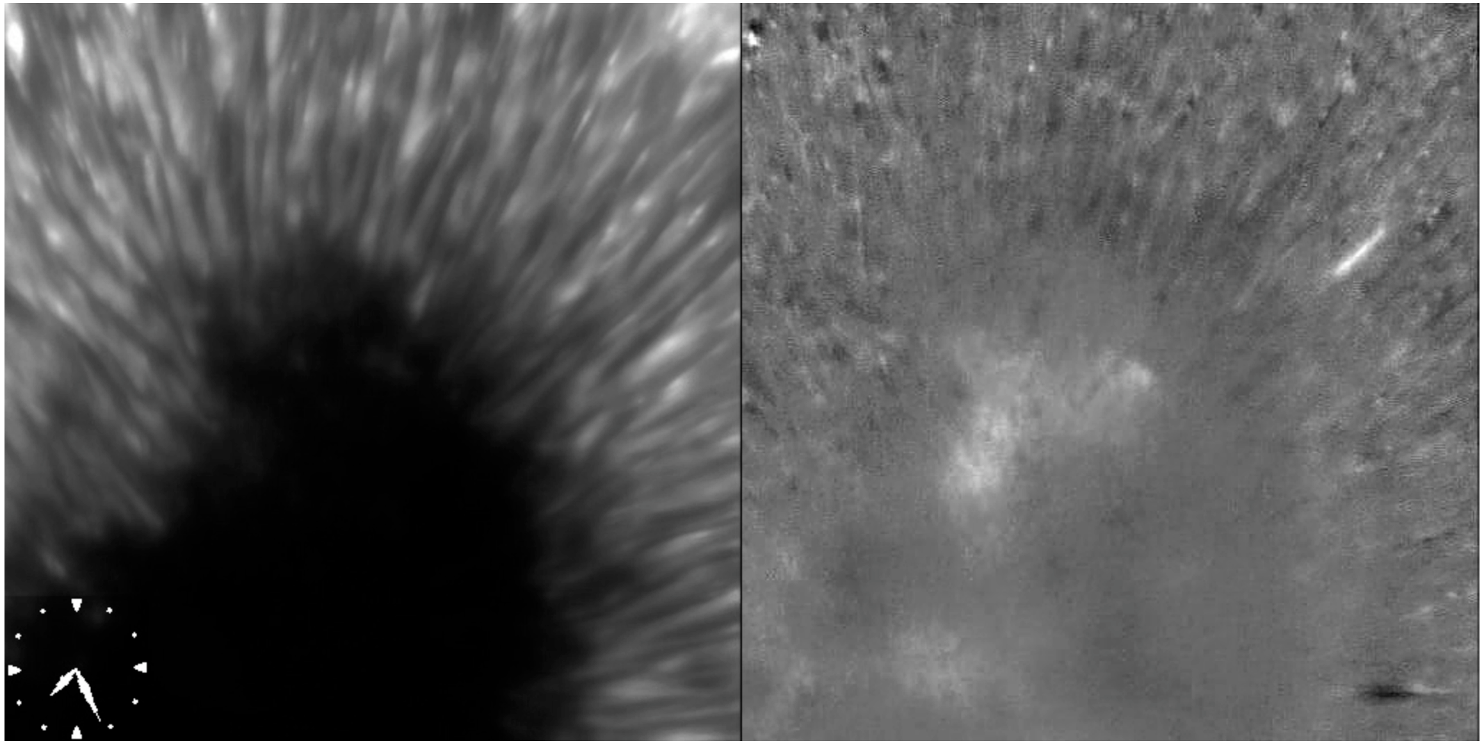
Solar penumbral microjet

Collimated ejection of a sunspot plasma

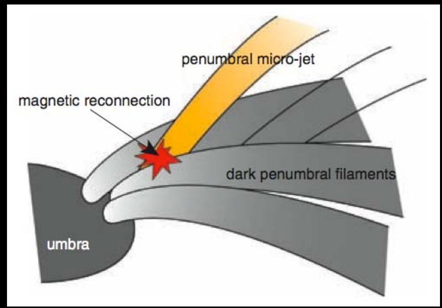
Observation of solar penumbral jets

Katsukawa et al. (2007)



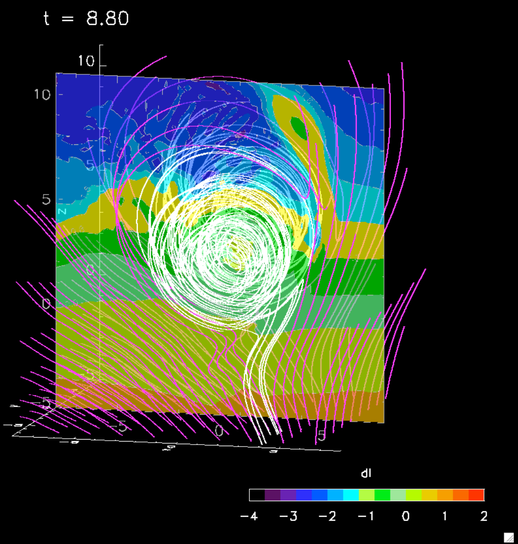
Observed by ***Hinode***

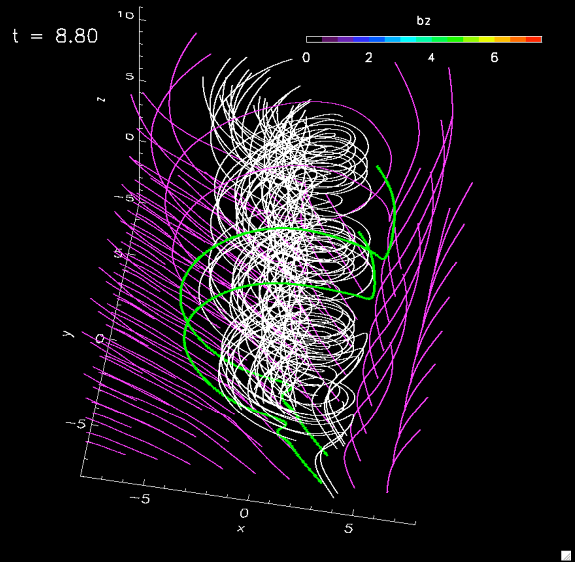
Schematic model of a solar penumbral jet



Katsukawa et al. (2007)

Numerical simulation of a solar penumbral jet

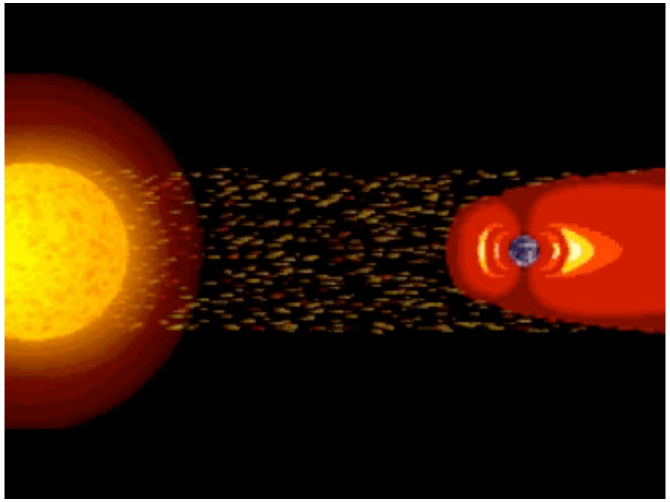




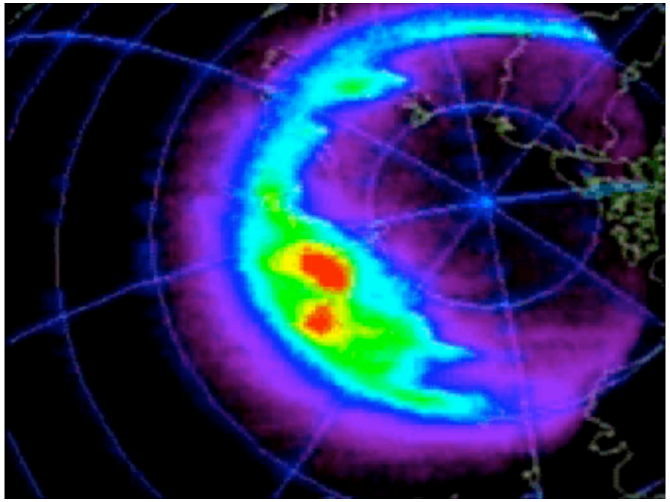
Magara (2010)

Interactions between a solar wind and a magnetosphere

Solar wind (schematic model)

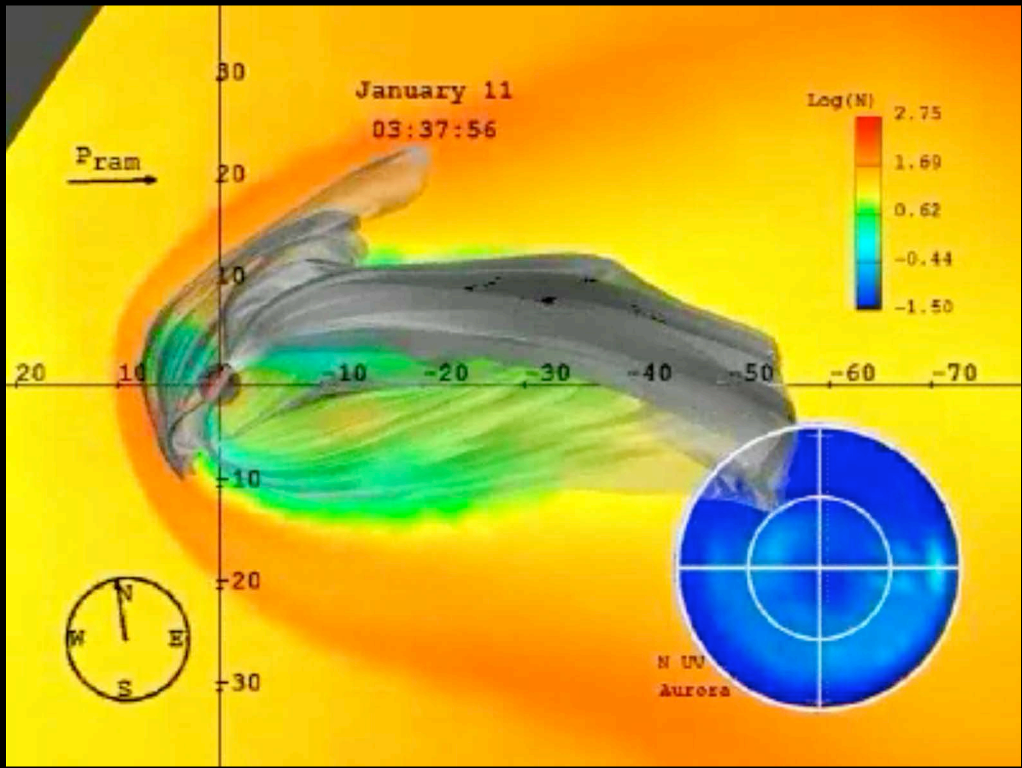


Aurora observed from **the space**



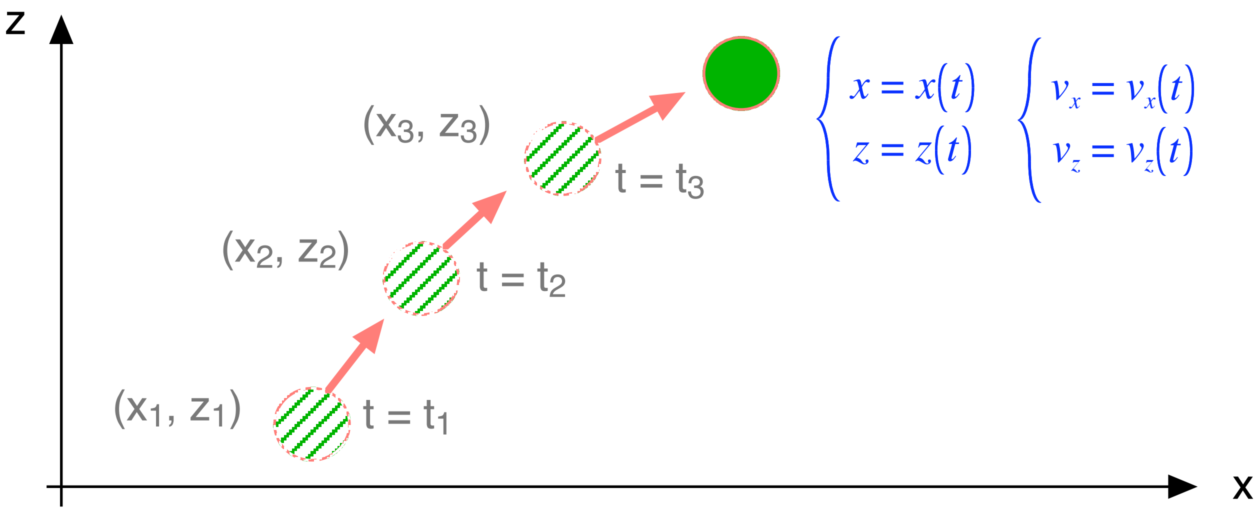
Aurora observed from **the ground**

Numerical simulation of a solar wind interacting with a magnetosphere



Comparison between mechanics and fluid dynamics

Mechanics



$$\begin{cases} x = x(t) \\ z = z(t) \end{cases} \qquad \begin{cases} v_x = v_x(t) \\ v_z = v_z(t) \end{cases}$$

In mechanics, we DO NOT consider the internal structure of an object, just considering its position and velocity.

The equation of mechanics is a differential equation where time is the only independent variable (**ordinary differential equation, ODE**).

$$\begin{cases} m\dfrac{dv_x}{dt} = F_x\big(x(t), z(t), v_x(t), v_z(t), t\big) \\ m\dfrac{dv_z}{dt} = F_z\big(x(t), z(t), v_x(t), v_z(t), t\big) \end{cases} \qquad \begin{cases} v_x(t) = \dfrac{dx}{dt} \\ v_z(t) = \dfrac{dz}{dt} \end{cases}$$

Fluid dynamics

In fluid dynamics, we DO consider the internal structure of an object, such as the distribution of temperature in the object, which depends on not only time but also position. The temperature is therefore expressed as a function of time and position:

$$T = f(x, z, t)$$

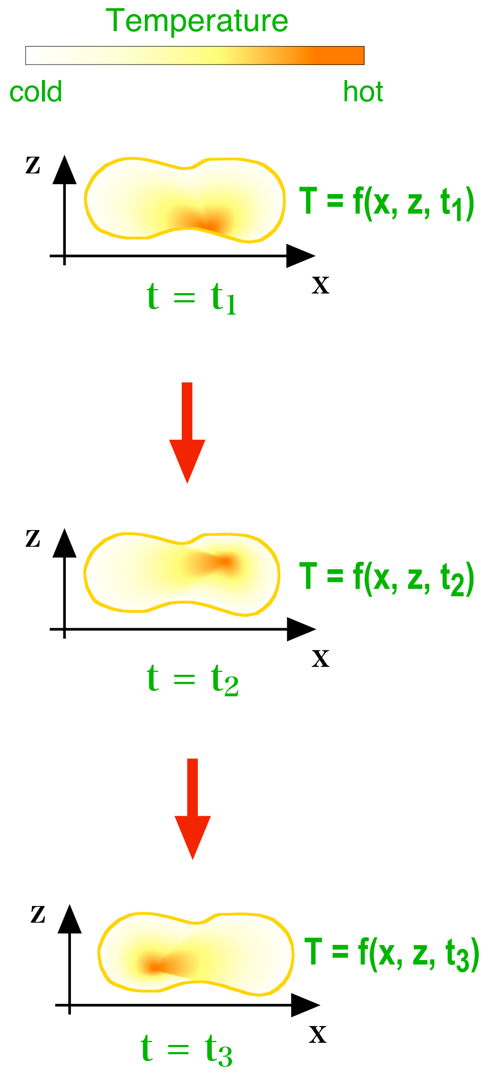
Temperature field

Field... continuously distributed in space-time

The equation for temperature field is given by a differential equation where time and position are independent variables.

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

... **partial differential equation**



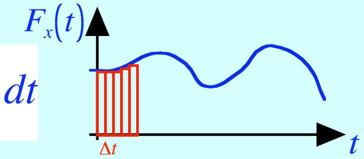
Integration of a differential equation (mechanics vs. fluid dynamics)

Mechanics

$$\frac{dv_x}{dt} = \frac{1}{m} F_x(t)$$ ... Ordinary differential equation (ODE)

Integration with respect to t leads to a solution.

$$v_x(t) = \int \frac{dv_x}{dt} dt = \frac{1}{m} \int F_x(t)\, dt$$



For numerical integration we introduce grids to time coordinate.
(**discretization**)

Fluid dynamics

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right)$$ ... Partial differential equation (PDE)

Integration with respect to t does not lead to a solution.

$$T(x, z, t) \neq \int \frac{\partial T}{\partial t} dt$$

Fluid dynamics

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right)$$ ... Partial differential equation (PDE)

Integration with respect to *t* does not lead to a solution.

$$T(x, z, t) \neq \int \frac{\partial T}{\partial t}\, dt$$

How can we numerically integrate a PDE?

↓

Introduce grids to space-time (x, y, z, t)

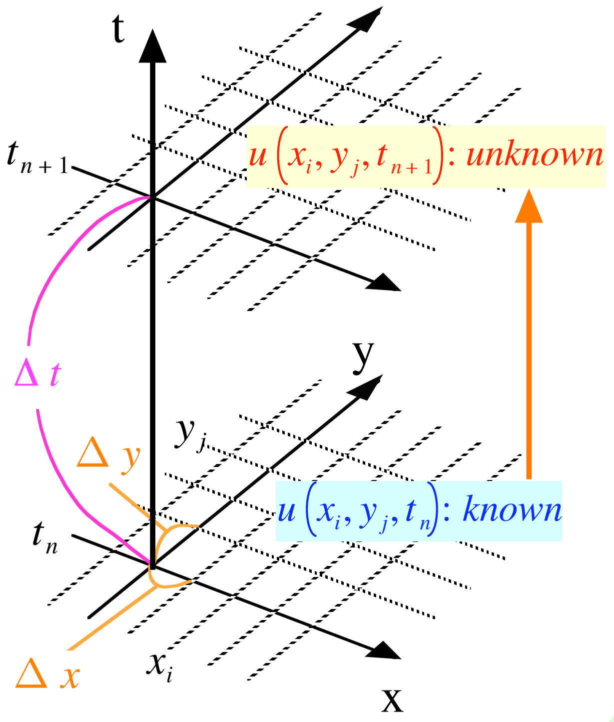
Discretization of space-time: introduce grids to space-time ($\Delta x, \Delta y, \Delta z, \Delta t$)

Differential equation → **Difference equation**

continuous space-time
(x, y, z, t)

discrete space-time
$(x_i, y_j, z_k, t_n): i, j, k, n = 1, 2, \ldots$

Discretization of 2D space-time



Discretization of 1D space-time

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

$$\Downarrow$$

$$\frac{\Delta u}{\Delta t} + a\frac{\Delta u}{\Delta x} = 0$$

$$\Downarrow$$

$$\frac{u(x_i, t_{n+1}) - u(x_i, t_n)}{t_{n+1} - t_n} + a\frac{u(x_{i+1}, t_n) - u(x_i, t_n)}{x_{i+1} - x_i} = 0$$

$$\Downarrow$$

$$u_i^{n+1} = u_i^n - a\frac{\Delta t}{\Delta x}\left(u_{i+1}^n - u_i^n\right)$$

unknown: u_i^{n+1} — known: $u_i^n - a\frac{\Delta t}{\Delta x}\left(u_{i+1}^n - u_i^n\right)$

$u_i^n = u(x_i, t_n)$
$(i = 1, 2, \ldots)$

Computer can calculate unknown values from known values via **four kinds of arithmetic operations** (**+**, **−**, **×**, **÷**).