Comparison between frozen-in evolution and diffusion

magnetic Reynolds number is a key non-dimensional parameter

$$\frac{\tau_{diff}}{\tau_{frozen}} \sim \frac{\dfrac{l_0^2}{\eta_{diff}}}{\dfrac{l_0}{v_0}} = \frac{v_0\, l_0}{\eta_{diff}} \equiv \mathrm{Rm}$$

$$\frac{E_{res}}{E_{conv}} \sim \frac{\eta_{diff} \dfrac{B_0}{l_0^2}}{v_0\, B_0} = \frac{\eta_{diff}}{v_0\, l_0} \equiv \mathrm{Rm}^{-1}$$

dimensional form

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left(\boldsymbol{v} \times \boldsymbol{B} - \eta_{diff} \nabla \times \boldsymbol{B} \right)$$

dimensionless form

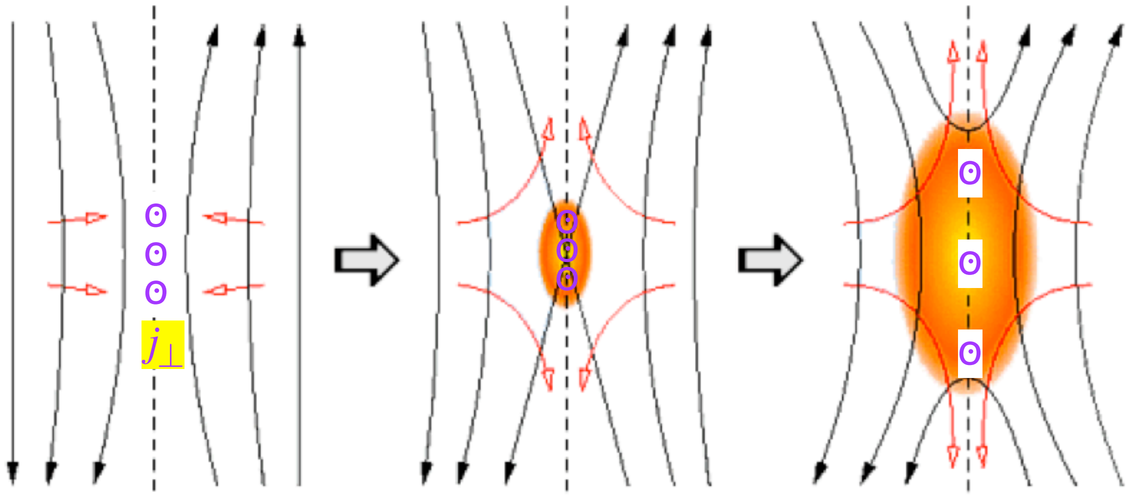
$$\rightarrow \quad \frac{\partial \boldsymbol{B}'}{\partial t'} = \nabla' \times \left(\boldsymbol{v}' \times \boldsymbol{B}' - \frac{1}{\mathrm{Rm}} \nabla' \times \boldsymbol{B}' \right)$$

Magnetic reconnection in MHD

evolution of magnetic field via both $\boldsymbol{E}_{conv}$ and $\boldsymbol{E}_{res}$

What is magnetic reconnection?

black arrows... **magnetic field**
red arrows ... **flow**
⊙... $j_\perp$ **(cross-field current)**



flow-coupled diffusion eq.

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) - \nabla \times (\eta_{diff} \nabla \times \boldsymbol{B})$$

$\boldsymbol{v} = \boldsymbol{0}$, uniform η_{diff} →

diffusion eq.

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta_{diff} \nabla^2 \boldsymbol{B}$$

It is **flow-coupled diffusion** by which **$j_\perp$-based free magnetic energy** is efficiently converted into thermal and kinetic energy.

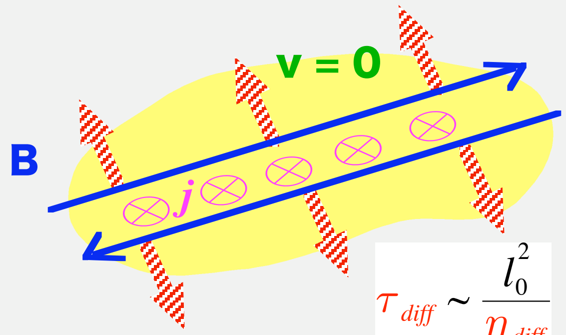
Difference between diffusion and reconnection...

- **Diffusion**

No flow (not dynamic process)

Magnetic field diffuses through a static plasma.

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta_{diff} \nabla^2 \boldsymbol{B}$$



$$\tau_{diff} \sim \frac{l_0^2}{\eta_{diff}}$$

- **Reconnection**

Flow exists (dynamic process).

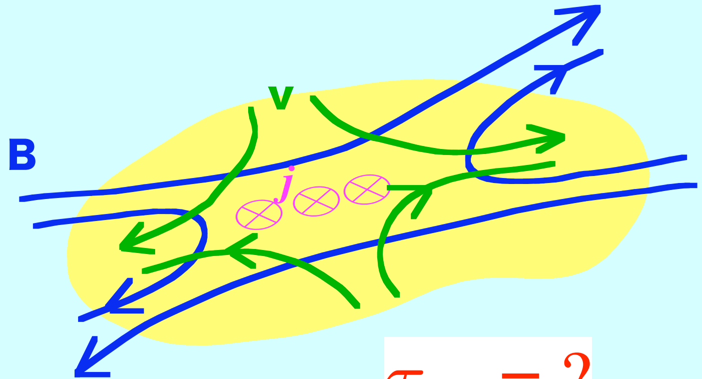
Magnetic field and flow interact with each other.

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \boldsymbol{v}) = 0$$

$$\rho \left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \bullet \nabla \boldsymbol{v} \right) = -\nabla p + \frac{1}{\mu_0} (\nabla \times \boldsymbol{B}) \times \boldsymbol{B}$$

$$\rho \frac{d}{dt} \left(\frac{1}{\gamma - 1} \frac{p}{\rho} \right) + p \nabla \bullet \boldsymbol{v} = \eta_{diff} \frac{|\nabla \times \boldsymbol{B}|^2}{\mu_0}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B} - \eta_{diff} \nabla \times \boldsymbol{B})$$



$$\tau_{rec} = ?$$

Sweet-Parker model

Model description

Approximations:

steady ($\frac{\partial}{\partial t} \sim 0$)

uniform density ($\rho \sim \rho_0$)

From the momentum equation (integrated),

In the x-direction

$$P_0 \sim \frac{B_0^2}{2\mu_0} \left(x = \pm \frac{l}{2} \right)$$ ... **pressure equilibrium**

diffusion region... high-β state
inflow region... low-β state

In the y-direction

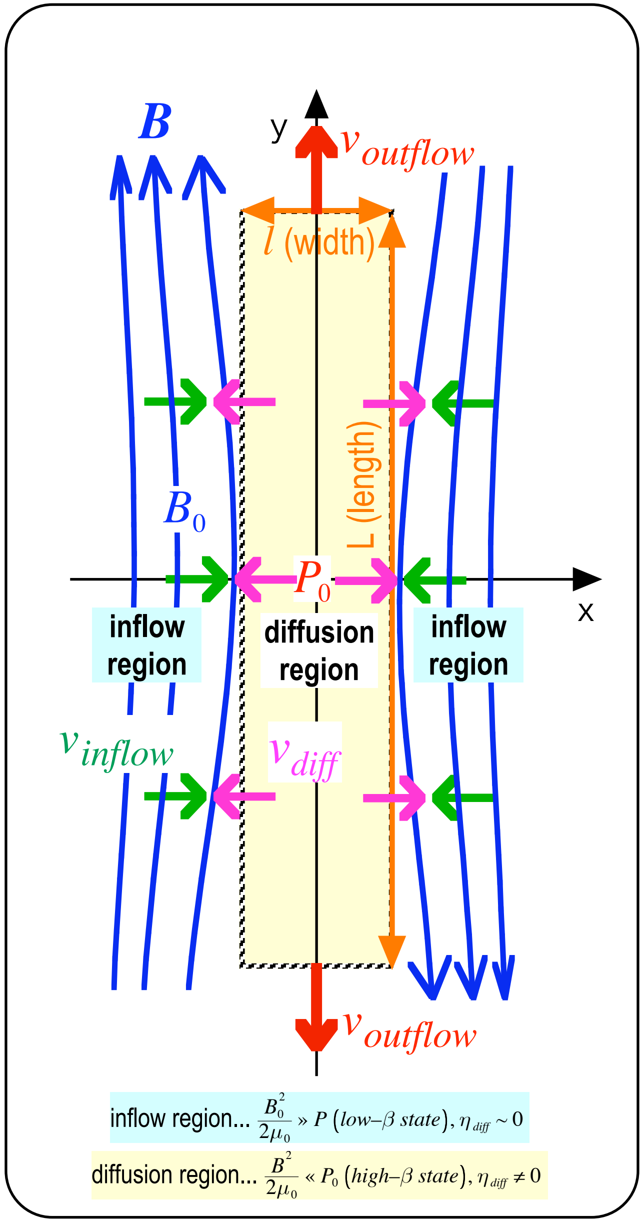
$$\frac{1}{2} \rho_0 v_y^2 (y=0) + P_0 = \frac{1}{2} \rho_0 v_{outflow}^2 + P(y = \pm 1/2\, L)$$ **... Bernoulli's theorem**

Here, $v_y(y=0) \sim 0$ and $P(y = \pm 1/2\, L) \ll \frac{1}{2}\rho_0 v_{outflow}^2$

$$\therefore P_0 \sim \frac{1}{2} \rho_0 v_{outflow}^2$$

$$v_{outflow} \sim \frac{B_0}{\sqrt{\mu_0 \rho_0}} \equiv v_{A0}$$

Outflow is accelerated by gas pressure gradient force and its speed is comparable to Alfvén speed.



From mass conservation (integrated),

$$\rho_0 v_{inflow} L = \rho_0 v_{outflow} l \sim \rho_0 v_{A0} l$$

$$\therefore v_{inflow} \sim v_{A0} \frac{l}{L}$$

From energy equation (integrated),

$$2 \times \frac{B_0^2}{2\mu_0} v_{inflow} L = \mu_0 \eta_{diff} j_0^2 l L$$ ... energy balance

Input energy — Released energy

$$\text{Here, } j_0 \sim \frac{1}{\mu_0} \frac{|-B_0 - B_0|}{l} = \frac{2}{\mu_0 l} B_0$$

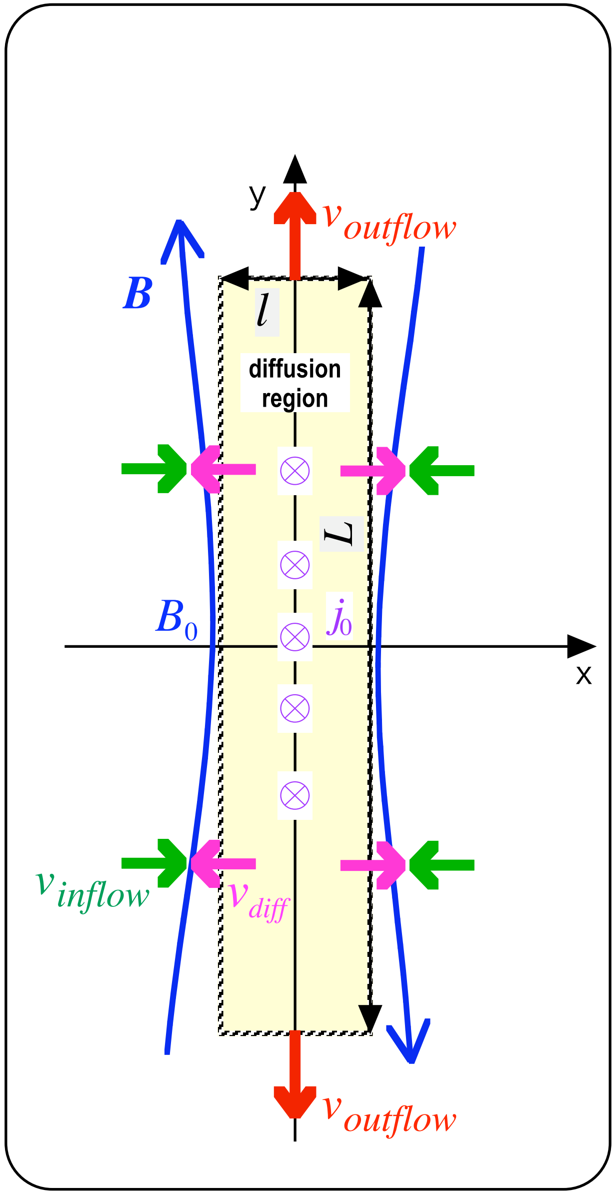
$$\therefore v_{inflow} \sim \frac{4 \eta_{diff}}{l}$$

Diffusion speed is given by

$$v_{diff} \sim \frac{\eta_{diff}}{l},$$

so inflow speed is balanced by diffusion speed (=> steady state).

$$v_{inflow} \sim v_{diff}$$



From mass conservation → $v_{inflow} \sim v_{A0} \dfrac{l}{L}$

From energy equation → $v_{inflow} \sim \dfrac{\eta_{diff}}{l}$

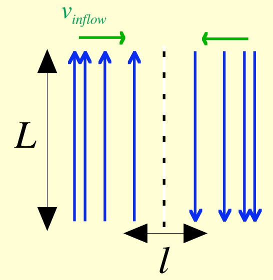
Here, we eliminate l, then v_{inflow} is given by

$$v_{inflow} \sim v_{A0} \frac{1}{L} \frac{\eta_{diff}}{v_{inflow}} \Rightarrow v_{inflow}^2 \sim v_{A0} \frac{\eta_{diff}}{L} = v_{A0}^2 \frac{\eta_{diff}}{v_{A0} L}$$

Reconnection speed in **Sweet-Parker model**

$$v_{inflow}^{SP} \sim v_{A0} R'^{-\frac{1}{2}}_{m}, \quad R'_{m} = \frac{v_{A0} L}{\eta_{diff}}$$

Lundquist number



Magnetic flux is continuously supplied to diffusion region by inflow, so the magnetic flux density is almost kept constant in the region. This is different from simple diffusion (flux density decreases with time).

Normalized time scale of **reconnection**

$$\frac{\tau_{rec}^{SP}}{\tau_{dyna}} = \frac{L/v_{inflow}}{L/v_{A0}} \sim R'^{\frac{1}{2}}_{m}$$

↔

Normalized time scale of **diffusion**

$$\frac{\tau_{diff}}{\tau_{dyna}} = \frac{\frac{L^2}{\eta_{diff}}}{\frac{L}{v_{A0}}} = \frac{v_{A0} L}{\eta_{diff}} = R'_{m}$$