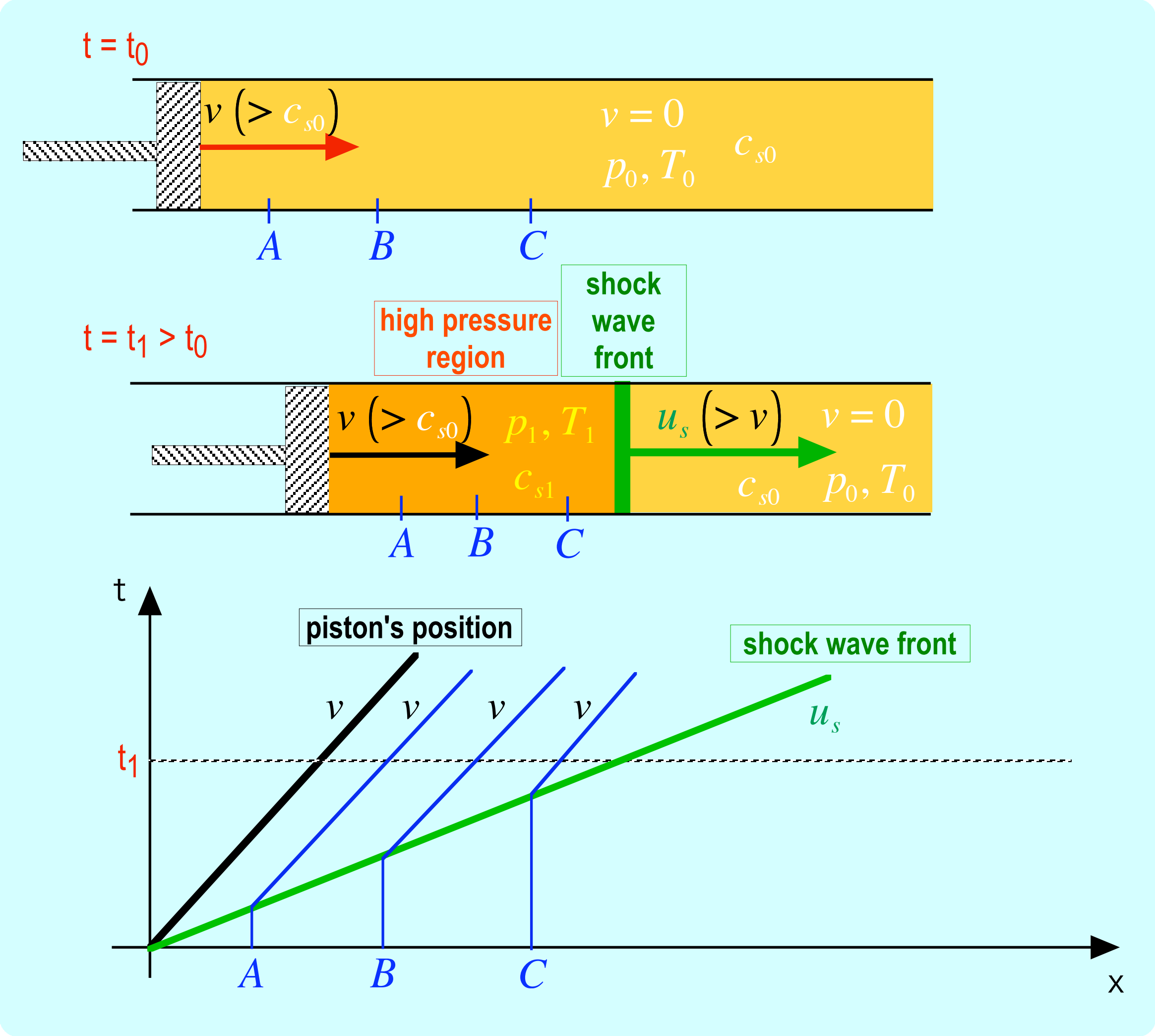
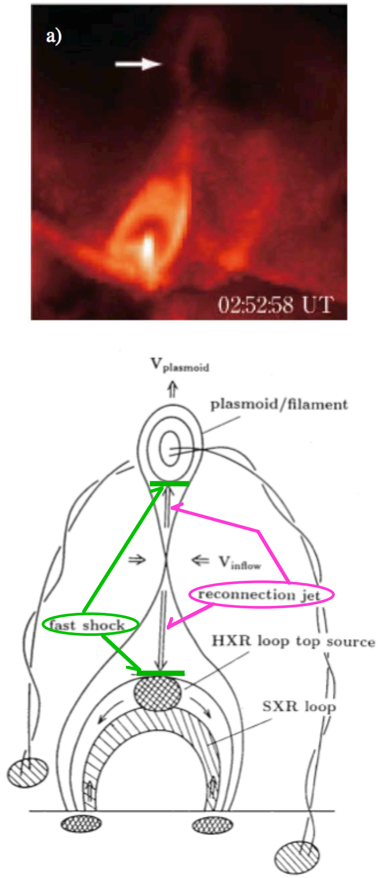
3) Kinetic energy is deposited... (piston-driven shock, jet)

associated with a **highly dynamic event**





4) Spontaneous enhancement of small-amplitude wave via nonlinear effect

associated with a **less dynamic event**

$$\rho\left(\frac{\partial}{\partial t}\boldsymbol{v} + \boldsymbol{v}\bullet\nabla\boldsymbol{v}\right) = \ldots$$

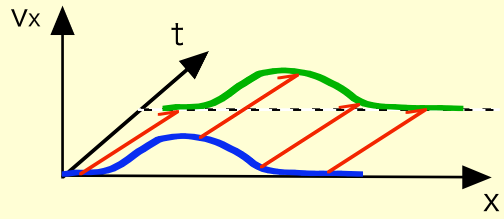
For a **linear wave**, we neglect this **nonlinear term**.

Consider an 1-dimensional case:

$$\frac{\partial}{\partial t}v_x + v_x\frac{\partial v_x}{\partial x} = 0$$

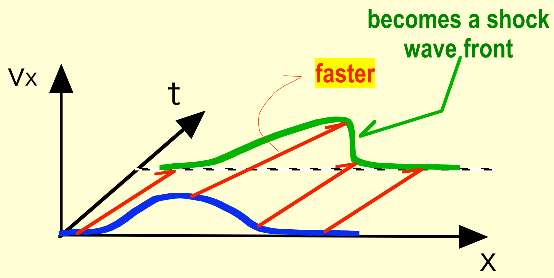
If this is constant, the profile of v_x is **conserved** (**linear wave**).

Each location propagates at the same speed.



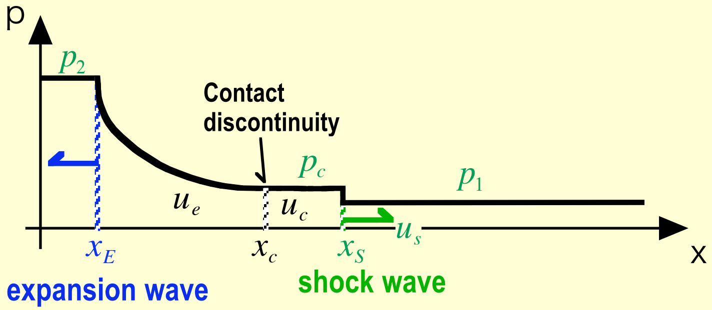
In fact, the propagation speed depends on the value of v_x, so the profile of v_x is **deformed** and **steepened** (**nonlinear effect**).

Each location propagates at different speed.



Analytic solutions of shock tube, blast wave, and piston-driven shock

shock tube



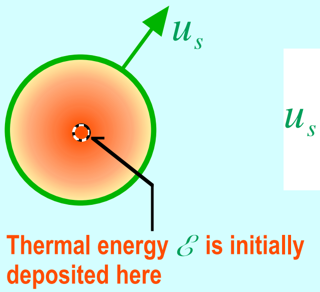
$$u_s = \frac{\gamma_1 + 1}{4} u_c + \sqrt{\frac{1}{16}\left(\gamma_1 + 1\right)^2 u_c^2 + c_1^2}$$

$$u_c = c_{s1}\left(\frac{p_c}{p_1} - 1\right)\sqrt{\frac{2/\gamma_1}{\left(\gamma_1 + 1\right) p_c/p_1 + \left(\gamma_1 - 1\right)}}$$

$$\frac{p_2}{p_1} = \frac{p_c}{p_1}\left[1 - \frac{\left(\gamma_2 - 1\right) c_{s1}/c_{s2} \left(p_c/p_1 - 1\right)}{\sqrt{2\gamma_1}\sqrt{2\gamma_1 + \left(\gamma_1 + 1\right)\left(p_c/p_1 - 1\right)}}\right]^{-2\frac{\gamma_2}{\gamma_2 - 1}}$$

(From *Elements of Gas Dynamics*)

blast wave



$$u_s \equiv \frac{dr_{sh}}{dt} = \frac{2}{5}\xi_0\left(\frac{\mathscr{E}}{\rho_1 t^3}\right)^{\frac{1}{5}}, \quad \xi_0 \sim o\left(1\right)$$

Sedov's solution

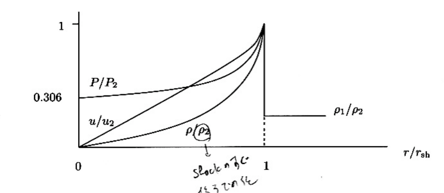
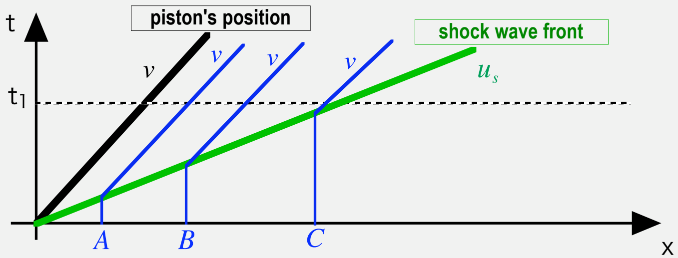


FIGURE 17.3
The blast-wave solution in units of the immediate postshock values for the case $\gamma = 5/3$.

(From *Gas Dynamics*)

piston-driven shock



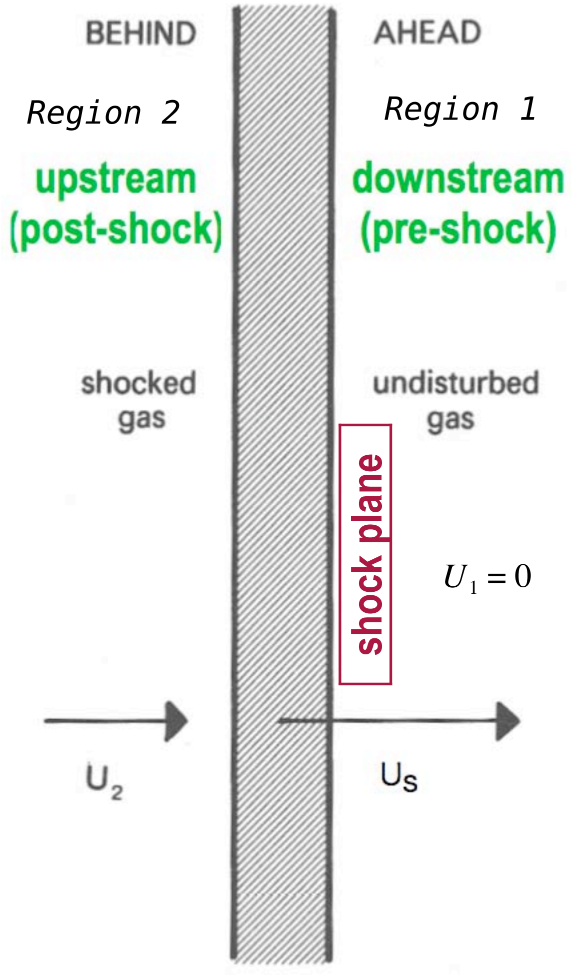
$$u_s = \frac{\gamma + 1}{4} v + \sqrt{\frac{1}{16}\left(\gamma + 1\right)^2 v^2 + c_s^2}$$

(From *Fluid Mechanics*)

Hydrodynamic shock wave

Perpendicular shock

... a plane shock wave propagating at a constant speed in the direction perpendicular to a shock plane



(From *Solar MHD*)

Convert a rest frame to a shock frame...

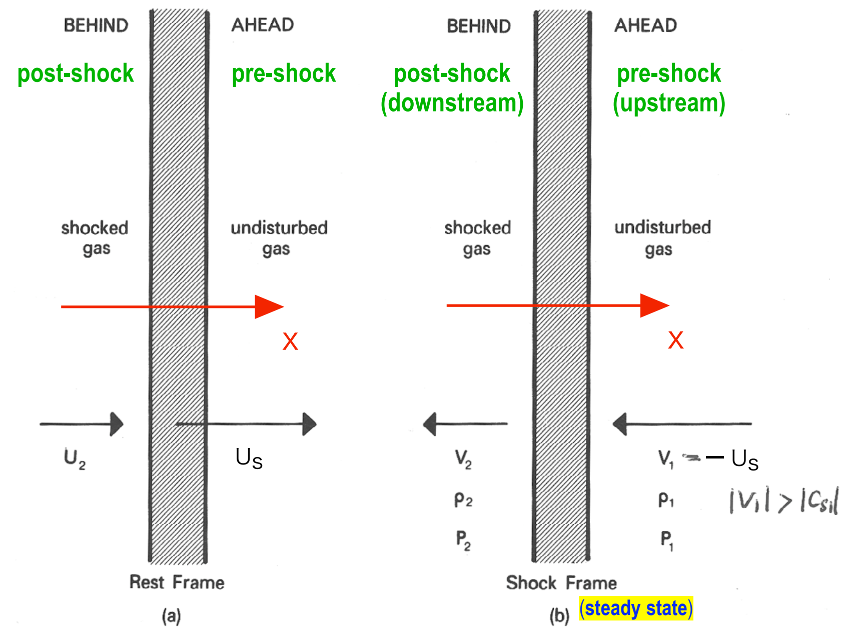


Fig. 5.3. The notation for a plane hydrodynamic shock wave moving to the right with speed u into a gas at rest. Properties ahead of the shock are denoted by 1 and those behind by 2.

(From *Solar MHD*)

Relation between the rest frame and shock frame (**steady state**):

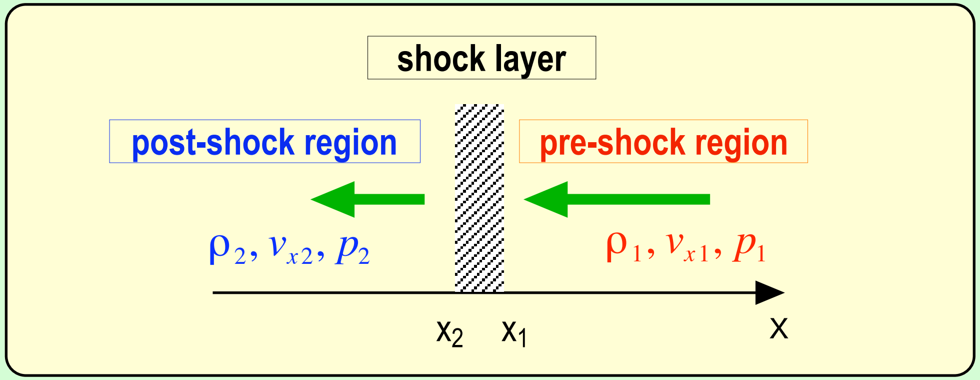
propagation speed of the shock wave is given by $v_{x1} = -U_S$

flow velocity in the post-shock region is given by $v_{x2} = v_{x1} + U_2 = -U_S + U_2$

In the shock frame, we consider three conservation laws (mass, momentum, energy) across a shock layer.

Mass conservation (1D, steady condition):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) = 0 \rightarrow \frac{d}{dx}(\rho v_x) = 0$$



$$\int_{x_2}^{x_1} \frac{d}{dx}(\rho v_x)\, dx = [\rho v_x]_{x_2}^{x_1} = \rho_1 v_{x1} - \rho_2 v_{x2} = 0$$

$$\longrightarrow \rho_1 v_{x1} = \rho_2 v_{x2}$$

Momentum conservation (1D, steady condition):

$$\frac{\partial}{\partial t}\left(\rho\, v_x\right) + \frac{\partial}{\partial x}\left(\rho\, v_x\, v_x + p\right) = \frac{\partial\, \pi_{xx}}{\partial x} \rightarrow \frac{d}{dx}\left(\rho\, v_x\, v_x + p\right) = \frac{d\, \pi_{xx}}{dx}$$

$$\int_{x_2}^{x_1} \frac{d}{dx}\left(\rho\, v_x\, v_x + p\right) dx = \left[\rho\, v_x^2 + p\right]_{x_2}^{x_1} = \rho_1\, v_{x\,1}^2 + p_1 - \left(\rho_2\, v_{x\,2}^2 + p_2\right) = \underbrace{\pi_{xx}\left(x_1\right)}_{=0} - \underbrace{\pi_{xx}\left(x_2\right)}_{=0}$$

because v is uniform just outside the shock layer

$$\longrightarrow \quad \rho_1\, v_{x\,1}^2 + p_1 = \rho_2\, v_{x\,2}^2 + p_2$$

Energy conservation (1D, steady condition):

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\,\rho\, v_x^2 + \frac{1}{\gamma - 1}\, p\right) + \frac{\partial}{\partial x}\left[\left(\frac{1}{2}\,\rho\, v_x^2 + \frac{1}{\gamma - 1}\, p + p\right) v_x\right] = q$$

$$\rightarrow \frac{d}{dx}\left[\left(\frac{1}{2}\,\rho\, v_x^2 + \frac{1}{\gamma - 1}\, p + p\right) v_x\right] = q$$

※ $$\int_{x_2}^{x_1} q\, dx = 0$$

$$q = \frac{\partial}{\partial x}\left(\pi_{xx} v_x - \mathscr{F}_x\right)$$

because v & T are uniform just outside the shock layer

$$\int_{x_2}^{x_1} \frac{d}{dx}\left[\left(\frac{1}{2}\,\rho\, v_x^2 + \frac{1}{\gamma - 1}\, p + p\right) v_x\right] dx = \left[\left(\frac{1}{2}\,\rho\, v_x^2 + \frac{\gamma}{\gamma - 1}\, p\right) v_x\right]_{x_2}^{x_1}$$

$$= \left(\frac{1}{2}\,\rho_1\, v_{x\,1}^2 + \frac{\gamma}{\gamma - 1}\, p_1\right) v_{x1} - \left(\frac{1}{2}\,\rho_2\, v_{x\,2}^2 + \frac{\gamma}{\gamma - 1}\, p_2\right) v_{x2} = 0$$

$$\longrightarrow \quad \left(\frac{1}{2}\,\rho_1\, v_{x\,1}^2 + \frac{\gamma}{\gamma - 1}\, p_1\right) v_{x\,1} = \left(\frac{1}{2}\,\rho_2\, v_{x\,2}^2 + \frac{\gamma}{\gamma - 1}\, p_2\right) v_{x2}$$