• $\rho_c \sim 0$ **(local charge neutrality)**

$$\nabla \bullet \boldsymbol{E} = \frac{\rho_c}{\varepsilon_0} \Rightarrow \frac{E_0}{l_0} \sim \frac{e\,(n_+ - n_-)}{\varepsilon_0}$$

$$n_+ - n_- \sim \frac{\varepsilon_0 E_0}{e\, l_0} \approx \frac{\varepsilon_0 \left(\frac{l_0 B}{t_0}\right)}{e\, l_0} = \frac{\varepsilon_0 v_0 B}{e\, l_0}$$

Condition of local charge neutrality:

$$|n_+ - n_-| << n_+ + n_- \equiv n_{total}$$ (total number density)

$$\therefore \frac{\varepsilon_0 v_0 B}{e\, l_0} << n_{total} \quad \longrightarrow \quad 6.7\times10^{7} \frac{v_0 B}{l_0} \ll n_{total}$$ (MKS unit)

$e = 1.6\times10^{-19}\, C$ (electron charge)

e.g. solar corona: $n_{total} \sim 10^{14}\ \mathrm{m}^{-3}$, $v_0 \sim 10^{5}$ m/s, $l_0 \sim 10^{7}$ m, $B \sim 10^{-2}$ T

$$6.7\times10^{7} \frac{v_0 B}{l_0} << n_{total} \quad \rightarrow \quad 6.7\times10^{3} << 10^{14}$$ ***... satisfied***

Coulomb's law $\nabla \bullet \boldsymbol{E} = \frac{\rho_c}{\varepsilon_0}$ is NOT used to determine electric field in MHD.

Ohm's law determines electric field in MHD from the state of plasma.

Magnetic energy density vs. electric energy density in MHD

Electric energy density... $\mathcal{E}_E = \dfrac{\varepsilon_0}{2} E^2$

Magnetic energy density... $\mathcal{E}_M = \dfrac{B^2}{2\mu_0}$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \longrightarrow \frac{E}{l_0} \approx \frac{B}{t_0}$$

The ratio is

$$\mathcal{E}_E / \mathcal{E}_M = \frac{\varepsilon_0 \mu_0 E^2}{B^2} \approx \frac{\left(\dfrac{l_0}{t_0} B\right)^2}{c^2 B^2} = \frac{v_0^2}{c^2} << 1$$

$\rightarrow$ electric energy density can be neglected in MHD

MHD induction equation

... represents the electromagnetic part

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$ **Faraday's law**

$$\boldsymbol{E}^{MHD} = -\boldsymbol{u}^{MHD} \times \boldsymbol{B} + \eta\, \boldsymbol{j}^{MHD}$$ **Ohm's law**

$\eta \equiv \frac{1}{\sigma}$ ... *resistivity*

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j}^{MHD}$$ **Ampere's law**

$\boldsymbol{E} = \boldsymbol{E}^{MHD}$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left(-\boldsymbol{u}^{MHD} \times \boldsymbol{B} + \eta\, \boldsymbol{j}^{MHD}\right)$$

eliminate $\boldsymbol{j}^{MHD}$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left(\boldsymbol{u}^{MHD} \times \boldsymbol{B} - \eta_{diff} \nabla \times \boldsymbol{B}\right)$$

$\eta_{diff} \equiv \frac{\eta}{\mu_0}$ ... *magnetic diffusivity*

MHD induction equation

... determines the evolution of magnetic field from the state of plasma

MHD equations

Fluid part

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \boldsymbol{u}) = 0$$ ... mass conservation (=> density)

$$\rho \left[\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \bullet \nabla \boldsymbol{u} \right] = - \nabla p + \nabla \bullet \overleftrightarrow{\pi} + \boldsymbol{j} \times \boldsymbol{B}$$ ... momentum equation (=> flow velocity)

$$\pi_{ik} = \mu D_{ik}, D_{ik} \equiv \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3} (\nabla \bullet \boldsymbol{u}) \delta_{ik} \text{ (deformation rate tensor)}$$

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \nabla \bullet (\rho \varepsilon \boldsymbol{u}) = - P \nabla \bullet \boldsymbol{u} - \nabla \bullet \mathscr{F} + \Psi + \eta j^2$$

$$\varepsilon = \frac{1}{\gamma - 1} \frac{P}{\rho}, \quad \gamma = \frac{5}{3} \qquad \mathscr{F}_i = - \kappa \frac{\partial T}{\partial x_i} \qquad \Psi \equiv \pi_{ik} \frac{\partial u_i}{\partial x_k}$$ ... internal energy equation (=> pressure)

$$T = \frac{P}{\rho} \frac{\bar{m}}{k_B}$$ ... equation of state (=> temperature)

Electromagnetic part

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} - \eta_{diff} \nabla \times \boldsymbol{B})$$ ... induction equation (=> magnetic field)

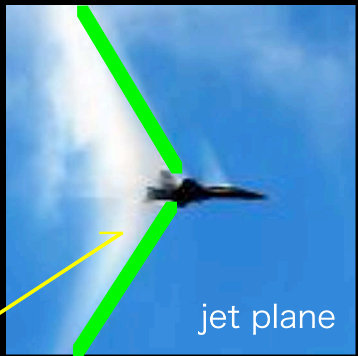
$$\boldsymbol{E} = - \boldsymbol{u} \times \boldsymbol{B} + \eta \boldsymbol{j}$$ ... Ohm's law (=> electric field)

$$\boldsymbol{j} = \frac{1}{\mu_0} \nabla \times \boldsymbol{B}$$ ... Ampere's law (=> current density)

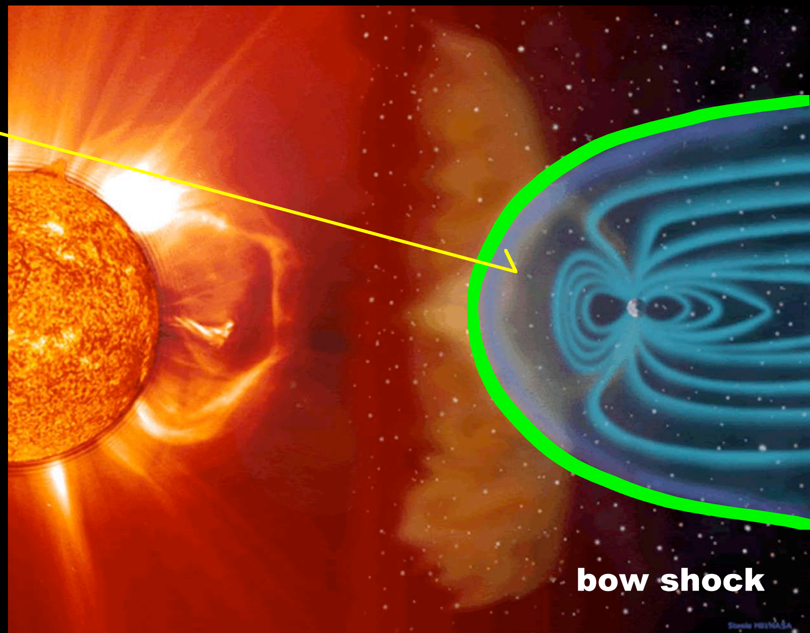
$$\nabla \bullet \boldsymbol{B} = 0$$ ... magnetic flux conservation (=> initial configuration of magnetic field)

Shock wave

Examples of shock waves



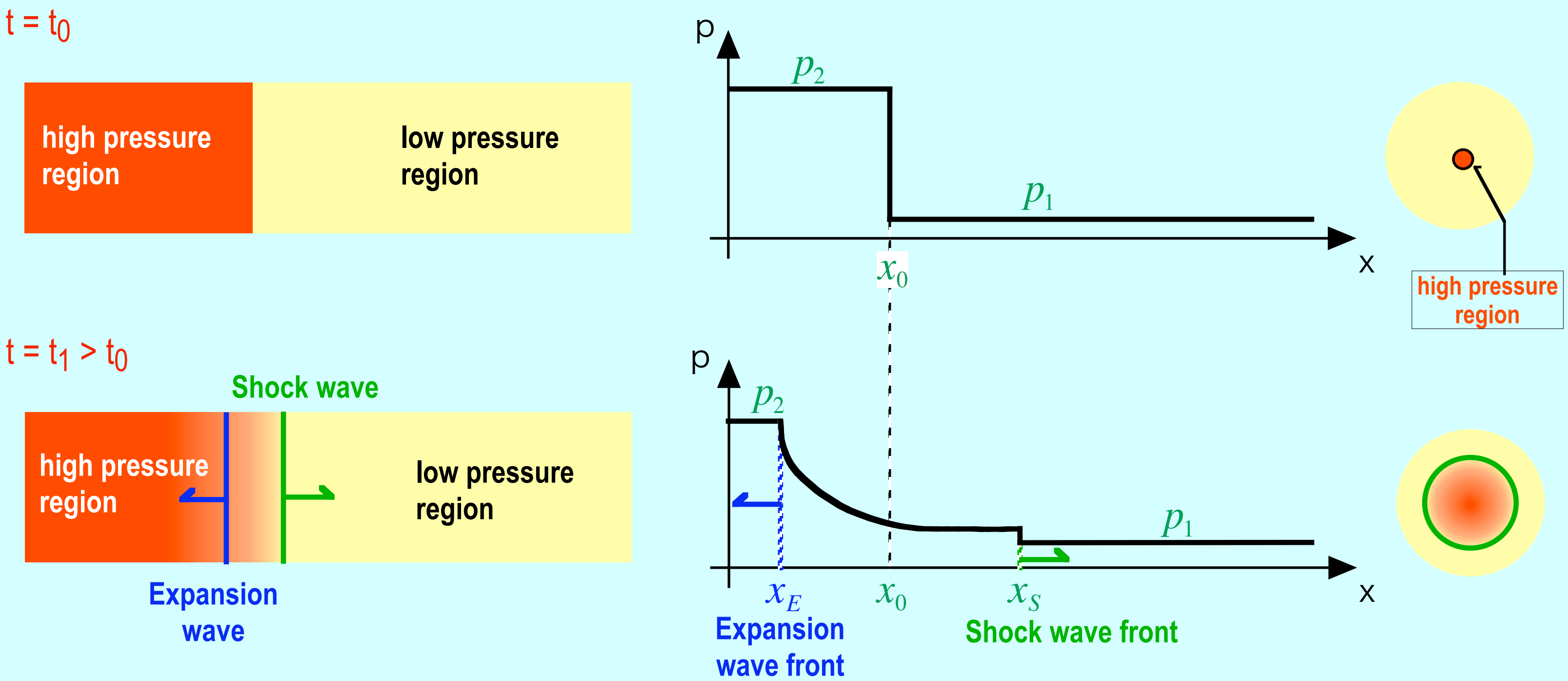
Behind a shock wave,
flow speed is lower than wave propagation speed;
density, pressure & temperature are enhanced



Generating mechanisms of a shock wave

1) Thermal energy is deposited... (shock tube, blast wave)

associated with a **highly dynamic event**

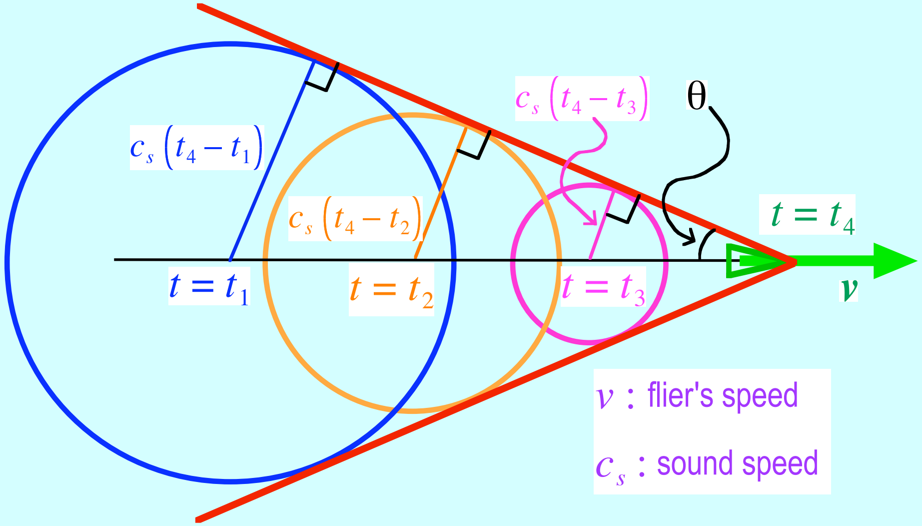


Toward the low-pressure region... **shock wave** propagates

Toward the high-pressure region... **expansion wave** propagates

2) Supersonic flier exists...

associated with a **highly dynamic event**



v : flier's speed

c_s : sound speed

A flier (green triangle) travels at a supersonic speed ($v > c_s$).

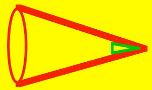
A sound wave generated at t_1, t_2 and t_3 forms a spherical wave front (blue, orange, pink).

The envelop of these spherical wave fronts forms a cone-shaped shock wave front (red).

The **inclination angle** ***θ*** is given by

$$v \sin\theta = c_s \quad \left(\leftrightarrow \frac{v}{c_s} \equiv M = \frac{1}{\sin\theta} > 1 \right)$$

When the Mach number M is **much larger than 1**, the shock wave front forms a **sharp cone** (small ***θ***).



When the Mach number M is **not much larger than 1**, the shock wave front is **detached from the flier** and forms an **arc-shaped bow shock** in front of the flier.