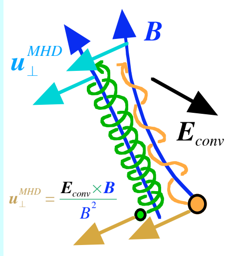
Three components of $E^{MHD} = E_{conv} + E_{res} + E_{Hall}$

All the components are associated with time variation in B

MHD electric field is generated by flow and current, while not by charge (due to local charge neutrality on MHD scale).
Faraday | Ohm + Hall | Coulomb

$E_{conv} \equiv -u^{MHD} \times B$ ... related to **average flow transverse to magnetic field**; caused by **gyration of protons and electrons around the magnetic field**

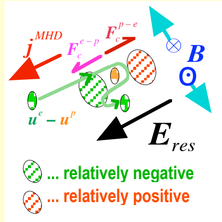
$B_{\perp}$–component



Convective electric field produces **transport of B** at the flow velocity $u_{\perp}^{MHD}$ **(froze-in)**. => charged particle gyrates around the same magnetic field line, or magnetic flux inside a gyration orbit is an invariant.
It causes energy conversion between magnetic energy and flow energy ($E_{conv} \bullet j^{MHD} = (j^{MHD} \times B) \bullet u^{MHD} \gtreqless 0$; **reversible process**).

$E_{res} \equiv \eta\, j^{MHD}$ ... related to **electric current**; caused by **collective collisions between protons and electrons**
(due to **charge imbalance via collective collisions**, which is different from **instantaneous Coulombic electric field** arising during a **collisional event between a proton and electron**)

$B_{\perp}, B_{//}$–components

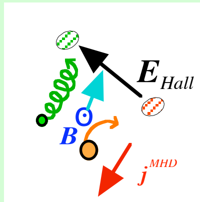


Resistive electric field produces **diffusion of B (non- frozen-in)**. => charged particles detach from magnetic field line. It causes energy conversion from magnetic energy to thermal energy ($E_{res} \bullet j^{MHD} > 0$; **irreversible process** => **Ohmic heating**). For astronomical & space plasmas, $\mathrm{Rm} \equiv \frac{v\,L}{\eta_{diff}} \sim \frac{E_{conv}}{E_{res}}$ is normally quite large, so E_{resis} is only effective in a local region where R_m becomes small (e.g. **current sheet**).

$$\frac{E_{res}}{E_{conv}} \sim \frac{1}{\mathrm{R}_m}$$

$E_{Hall} \equiv \frac{1}{e\,n}\, j^{MHD} \times B$ ... related to **electric current**; caused by **decoupling between protons and gyrating electrons** in $B_{\perp}$–*plane*

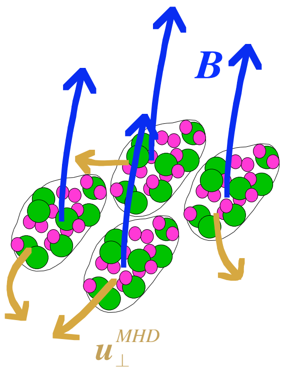
$B_{\perp}$–component



Hall electric field produces **slipping of B** (different from **diffusion**). => electrons and magnetic field slip through protons. It does not convert magnetic energy ($E_{Hall} \bullet j^{MHD} = 0$). It is effective in a local region where B is weak so that protons and electrons are decoupled in $B_{\perp}$–*plane* (e.g. central part of a **current sheet**).

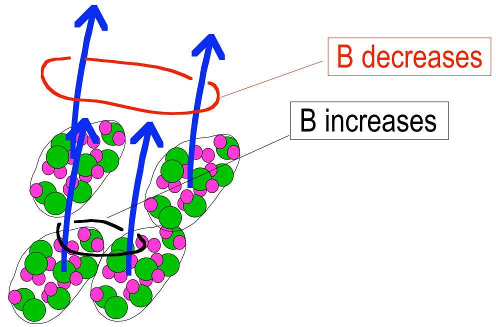
$$\frac{E_{Hall}}{E_{conv}} \sim \frac{t_B}{t_{MHD}} \qquad t_B \sim \omega_B^{-1} = \frac{M}{e\,B},\ t_{MHD} \sim \frac{L}{v_A} \quad \left(v_A = \frac{B}{\sqrt{\mu_0\,\rho}}\right)$$

time-varying B

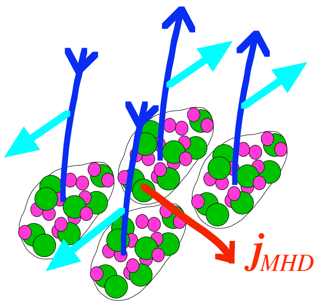


$$\boldsymbol{E}_{conv} \equiv -\boldsymbol{u}^{MHD} \times \boldsymbol{B}$$

convection
(frozen-in to MHD fluid element)

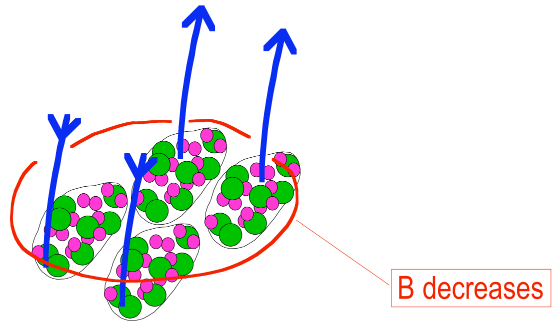


collisions between protons and electrons are less effective

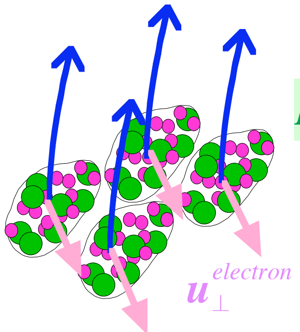


$$\boldsymbol{E}_{res} \equiv \eta\, \boldsymbol{j}^{MHD}$$

diffusion
(non-frozen-in)



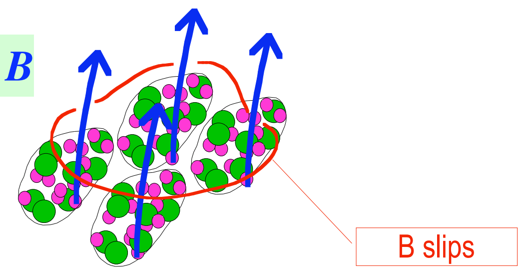
collisions between protons and electrons are effective



$$\boldsymbol{E}_{Hall} \equiv \frac{1}{e\,n}\, \boldsymbol{j}^{MHD} \times \boldsymbol{B} = -\boldsymbol{u}_{\perp}^{electron} \times \boldsymbol{B}$$

$\boldsymbol{u}_{\perp}^{proton} = 0$ (proton frame)

slipping
(frozen-in to electron fluid element)



collisions between protons and electrons are less effective