3) Ohm's law

Subtract **electron's momentum equation x M** from **proton's momentum equation x m** (nonlinear & viscous terms are not considered)

$$\frac{M\,m\,n}{e}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{j}^{MHD}}{n}\right) = e\,\rho^{MHD}\boldsymbol{E} + (M+m)\,\boldsymbol{F}_c^{p-e} - m\nabla P^p + M\nabla P^e + e\,n\left(m\,\boldsymbol{u}^p + M\,\boldsymbol{u}^e\right)\times\boldsymbol{B}$$

$-\boldsymbol{F}_c^{p-e} \propto \boldsymbol{j}^{MHD}$ $(both \propto \boldsymbol{u}^p - \boldsymbol{u}^e)$

$$\boldsymbol{F}_c^{p-e} = -n\,e\,\eta\,\boldsymbol{j}^{MHD}$$

$$\eta = \frac{M}{n\,e^2\,t_c^{p-e}} = \frac{m}{n\,e^2\,t_c^{e-p}}$$: resistivity

$$\eta_{diff} \equiv \frac{1}{\mu_0}\eta = \frac{c^2\,t_p^2}{t_c^{e-p}}$$: magnetic diffusivity

MKS unit

$$m\,\boldsymbol{u}^p + M\,\boldsymbol{u}^e = M\,\boldsymbol{u}^p + m\,\boldsymbol{u}^e + M\left(\boldsymbol{u}^e - \boldsymbol{u}^p\right) + m\left(\boldsymbol{u}^p - \boldsymbol{u}^e\right)$$
$$= \frac{\rho^{MHD}}{n}\boldsymbol{u}^{MHD} - (M-m)\frac{\boldsymbol{j}^{MHD}}{ne}$$

$$\boldsymbol{E} = -\boldsymbol{u}^{MHD}\times\boldsymbol{B} + \eta\,\boldsymbol{j}^{MHD} + \frac{M}{e\,\rho^{MHD}}\left[\frac{m\,n}{e}\frac{\partial}{\partial t}\left(\frac{\boldsymbol{j}^{MHD}}{n}\right) + \left(1-\frac{m}{M}\right)\boldsymbol{j}^{MHD}\times\boldsymbol{B} + \nabla\left(\frac{m}{M}P^p - P^e\right)\right]$$

approximation   approximation   approximation

They are all volume force:

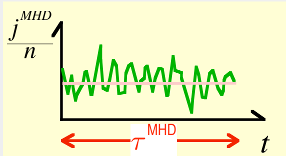
$m\,n\,\frac{\partial(\boldsymbol{u}^p - \boldsymbol{u}^e)}{\partial t}$ { typical time scale: τ^{rel}; typical time scale: l^{rel} }

$\boldsymbol{j}^{MHD}\times\boldsymbol{B}$ { typical time scale: τ^{MHD}; typical time scale: l^{MHD} }

$-\nabla P^e$ { typical time scale: τ^{e}; typical time scale: l^{e} }

assumption & approximation

assumption: $\frac{\partial}{\partial t}\left(\frac{\boldsymbol{j}^{MHD}}{n}\right) \sim 0 \Rightarrow \frac{\partial}{\partial t}\left(\boldsymbol{u}^p - \boldsymbol{u}^e\right) \sim 0$ *on MHD time scale*



approximation: $\frac{m}{M} << 1$

$$\boldsymbol{E}^{MHD} = -\boldsymbol{u}^{MHD}\times\boldsymbol{B} + \eta\,\boldsymbol{j}^{MHD} + \frac{1}{e\,n}\left[\boldsymbol{j}^{MHD}\times\boldsymbol{B} - \nabla P^e\right]$$

assumption: $-\nabla P^e \sim 0$

Electron's inertia is small enough to remove the nonuniformity of electron's pressure during τ^{MHD}.

Ohm's law in MHD

determines electric field from the state of plasma