From Boltzmann's equation to fluid dynamics equations (mathematical derivation)

Integrate it with χ in velocity space:

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \bullet \frac{\partial f}{\partial \boldsymbol{x}} + \frac{\boldsymbol{F}}{m} \bullet \frac{\partial f}{\partial \boldsymbol{v}} = \left(\frac{\delta f}{\delta t}\right)_C \longrightarrow \int_v \chi(v_i) \left(\frac{\partial f}{\partial t} + v_k \frac{\partial f}{\partial x_k} + \frac{F_k}{m}\frac{\partial f}{\partial v_k}\right) d^3v = \int_v \chi(v_i) \left(\frac{\delta f}{\delta t}\right)_c d^3v, \quad \chi(\boldsymbol{v}) = m, m\boldsymbol{v}, \frac{1}{2}mv^2$$

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k = 1... x-component; k = 2... y-component; k = 3... z-component

Left-hand side => **partial integral of each term**

$$\int_v \frac{\partial}{\partial t}(\chi f)\, d^3v + \int_v \left[\frac{\partial}{\partial x_k}(\chi\, v_k\, f) - f\frac{\partial}{\partial x_k}(\chi\, v_k)\right] d^3v + \int_v \left[\frac{\partial}{\partial v_k}\left(\chi\, f\, \frac{F_k}{m}\right) - f\frac{\partial}{\partial v_k}\left(\chi\,\frac{F_k}{m}\right)\right] d^3v$$

$\frac{\partial \chi}{\partial t} = 0 \; (\chi = \chi(v_i))$

$= 0 \; because \; \chi = \chi(v_i)$

$\to 0 \; because \; f \to 0 \; when \; v_i \to \pm\infty$

$$= \frac{\partial}{\partial t}(n\langle\chi\rangle) + \frac{\partial}{\partial x_k}(n\langle\chi\, v_k\rangle) - \left(n\left\langle\frac{F_k}{m}\frac{\partial\chi}{\partial v_k}\right\rangle\right)$$

$\int_v \Diamond\, f\, d^3v = \langle\Diamond\rangle \int_v f\, d^3v = n\,\langle\Diamond\rangle$

$\frac{\partial F_k}{\partial v_k} = 0$ for $gravity, Coulomb\ force, Lorentz\ force$

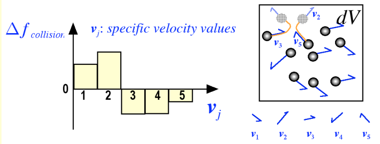
Right-hand side => **variation in χ via collision (perfect elastic collision)**

$$\int_v \chi(\boldsymbol{v})\left(\frac{\delta f(\boldsymbol{v})}{\delta t}\right)_c d^3v \approx \frac{1}{\Delta t}\int_v \chi(\boldsymbol{v})\, \Delta f_{collision}(\boldsymbol{v})\, d^3v$$

$$\frac{1}{\Delta t}\sum_{j=1}^{N} m\,\Delta f_{collision}(\boldsymbol{v}_j)$$

$$\frac{1}{\Delta t}\sum_{j=1}^{N} m\,\boldsymbol{v}_j \Delta f_{collision}(\boldsymbol{v}_j)$$

$$\frac{1}{\Delta t}\sum_{j=1}^{N} \frac{1}{2} m\, v_j^2 \Delta f_{collision}(\boldsymbol{v}_j)$$



$\int_v m\left(\frac{\delta f}{\delta t}\right)_c d^3v = 0$ ... mass conservation via collision in dV *(position space)* for single/multiple species system

$\int_v m\boldsymbol{v}\left(\frac{\delta f}{\delta t}\right)_c d^3v = \boldsymbol{0}$ ... momentum conservation via collision in dV *(position space)* for single species system (**for multiple species system, not conserved**)

$\int_v \frac{1}{2}mv^2\left(\frac{\delta f}{\delta t}\right)_c d^3v = 0$ ... kinetic energy conservation via collision in dV *(position space)* for single species system (**for multiple species system, not conserved**)

For single species system, $\int_v \chi(\boldsymbol{v}) \left(\frac{\delta f}{\delta t}\right)_c d^3 v = 0 \;\; when \;\; \chi(\boldsymbol{v}) = m, m\boldsymbol{v}, \frac{1}{2}mv^2$ **(all are conserved)**

$$\frac{\partial}{\partial t}\left(n\left\langle\chi\right\rangle\right)+\frac{\partial}{\partial x_k}\left(n\left\langle\chi\, v_k\right\rangle\right)-\left(n\left\langle\frac{F_k}{m}\frac{\partial\chi}{\partial v_k}\right\rangle\right)=0 \quad \ldots \text{※}$$