

In the case of plasmas,

$$\mathbf{F} = q \left(\mathbf{E}^* + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \dots \text{electromagnetic force (CGS unit)}$$

q : charge of a particle

| MKS | CGS |
|-------------------|---------------------------|
| B | $\frac{B}{c}$ |
| ϵ_0^{-1} | 4π |
| μ_0 | $\frac{4\pi}{c^2}$ |
| e | $e \sqrt{4\pi\epsilon_0}$ |

$$\longrightarrow \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m} \left(\mathbf{E}^* + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\delta f}{\delta t} \right)_c^{**}$$

Regarding the electric field*, it is different from instantaneous electric field that arises during an event of Coulombic collision (\Rightarrow particle-particle interaction via the instantaneous electric field).

Regarding the collision term**, we may consider wave-particle interaction (anomalous collision) as well as particle-particle interaction.

If collision can be neglected ($t_{\text{phenomenon}} \ll t_{\text{collision}}$),

$$\longrightarrow \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m} \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

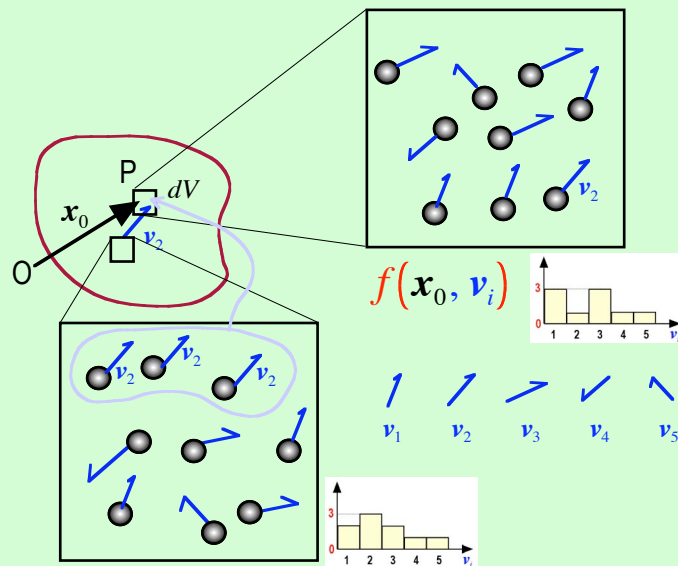
**Collisionless Boltzmann's equation
or
Vlasov equation**

Physical interpretation of Boltzmann's equation

We consider what causes temporal change of the distribution function at $(\mathbf{x}_0, \mathbf{v}_2)$:

$$\frac{\partial f(\mathbf{x}_0, \mathbf{v}_2, t)}{\partial t} = -\mathbf{v}_2 \cdot \frac{\partial f(\mathbf{x}, \mathbf{v}_2, t)}{\partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_0} - \frac{\mathbf{F}}{m} \cdot \frac{\partial f(\mathbf{x}_0, \mathbf{v}, t)}{\partial \mathbf{v}} \Big|_{\mathbf{v}=\mathbf{v}_2} + \left(\frac{\delta f}{\delta t} \right)_c$$

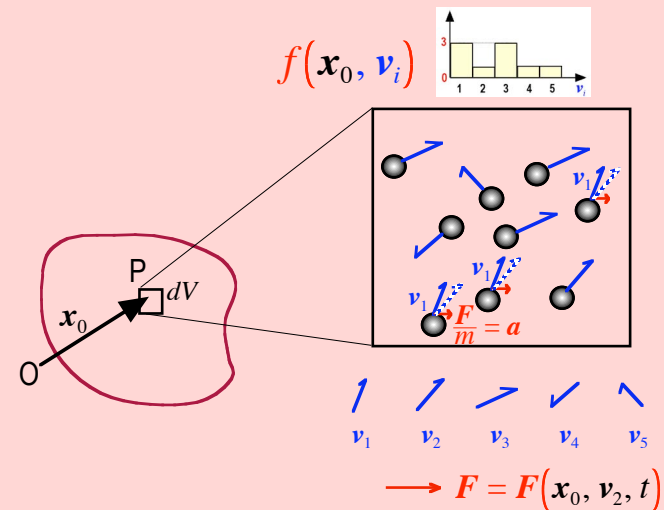
1st term: **flow effect** $-\mathbf{v}_2 \cdot \frac{\partial f(\mathbf{x}, \mathbf{v}_2, t)}{\partial \mathbf{x}} \Big|_{\mathbf{x}=\mathbf{x}_0}$



In this case, $f(\mathbf{x}_0, \mathbf{v}_2, t)$ increases with time.

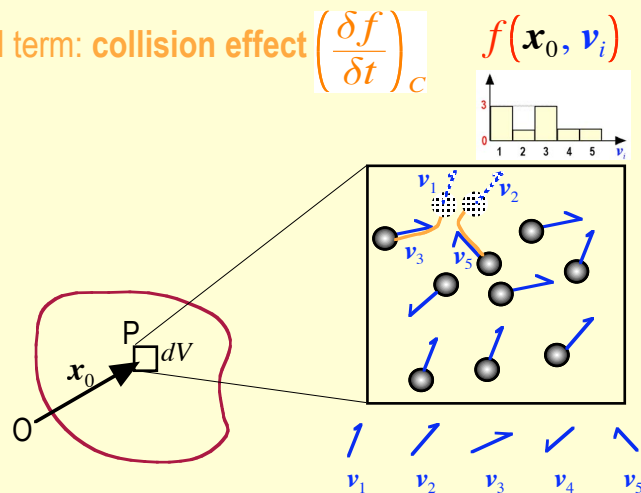
2nd term: **force effect** $-\frac{\mathbf{F}}{m} \cdot \frac{\partial f(\mathbf{x}_0, \mathbf{v}, t)}{\partial \mathbf{v}} \Big|_{\mathbf{v}=\mathbf{v}_2}$

\mathbf{F} is determined by external region of P.



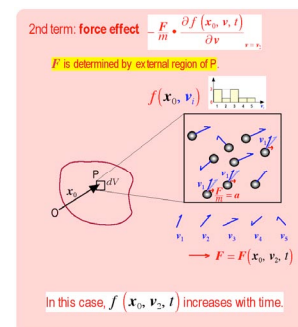
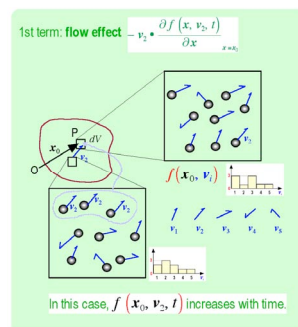
In this case, $f(\mathbf{x}_0, \mathbf{v}_2, t)$ increases with time.

3rd term: collision effect $\left(\frac{\delta f}{\delta t}\right)_c$

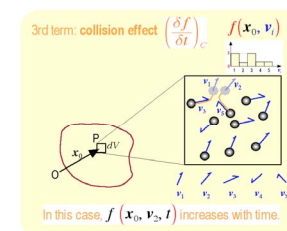


In this case, $f(x_0, v_2, t)$ increases with time.

$$\frac{\partial f(x_0, v_2, t)}{\partial t} = -v_2 \cdot \frac{\partial f(x, v_2, t)}{\partial x} \Big|_{x=x_0} - \frac{F}{m} \cdot \frac{\partial f(x_0, v, t)}{\partial v} \Big|_{v=v_2} + \left(\frac{\delta f}{\delta t}\right)_c$$



External origins



Internal origin
↓
source term

Average of Boltzmann's equation based on Maxwellian distribution function

from particle to fluid element