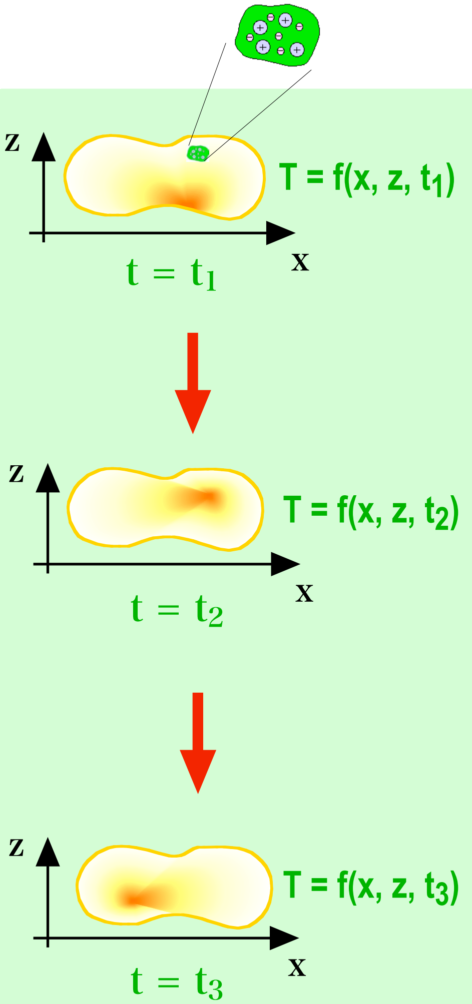
Fluid approach... Focus on mass density, flow velocity, pressure, temperature of a **fluid element** in space-time (these are statistically averaged quantities derived from position & velocity distributions of particles)

=> **Fluid dynamics equations**



In the fluid approach, we consider the **physical state** of a **fluid element** represented by **statistically averaged quantities** such as **density**, **flow velocity**, **pressure**, and **temperature**. For example, **temperature** changes with **position** and **time**, so it is expressed as a **function of position** and **time**:

$$T = f(x, z, t)$$

Temperature field

continuously distributed in space-time

Particles do not fill up the space, whereas fluid elements do that.

An equation for the temperature field is a **differential equation** where **position** and **time** are **independent variables** (partial differential equation, PDE).

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

Kinetic approach

(in the case of mechanics)

$$\begin{cases} m\dfrac{dv_x}{dt} = F_x\big(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t\big) \\ m\dfrac{dv_y}{dt} = F_y\big(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t\big) \\ m\dfrac{dv_z}{dt} = F_z\big(x(t), y(t), z(t), v_x(t), v_y(t), v_z(t), t\big) \end{cases} , \quad \begin{cases} \dfrac{dx}{dt} = v_x(t) \\ \dfrac{dy}{dt} = v_y(t) \\ \dfrac{dz}{dt} = v_z(t) \end{cases}$$

× N (number of particles)

... 6N ordinary differential equations
+ Maxwell's equations

Fluid approach

(in the case of magnetohydrodynamics)

$\dfrac{\partial \rho}{\partial t} + \nabla \bullet (\rho \boldsymbol{v}) = 0$ ... for ρ

$\rho\left(\dfrac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \bullet \nabla \boldsymbol{v}\right) = -\nabla p + \dfrac{1}{\mu_0}(\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \boldsymbol{F}$ ... for v_x, v_y, v_z

$\dfrac{\partial}{\partial t}\left(\dfrac{p}{\gamma - 1}\right) + \nabla \bullet \left(\dfrac{p}{\gamma - 1}\boldsymbol{v}\right) = -p\nabla \bullet \boldsymbol{v} + \nabla \bullet (\kappa_c \nabla T) + \dfrac{j^2}{\sigma}$ ... for P

$\dfrac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left(\boldsymbol{v} \times \boldsymbol{B} - \eta_{diff} \nabla \times \boldsymbol{B}\right)$ ... for B_x, B_y, B_z

... 8 partial differential equations
(+ equation of state)

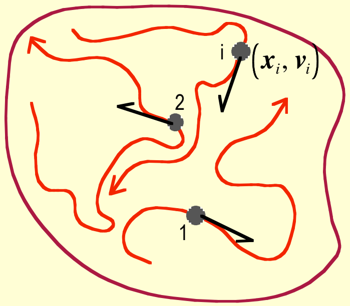
Because of local charge neutrality $\rho_c \sim 0$, Coulombic electric field does not exist globally (but electric field associated with time-varying magnetic field globally exists).

Since electric current globally exists while keeping the local charge neutrality, magnetic field globally exists.

Three types of dynamic systems (depend on the total number of particles N)

I. Small N system (discrete system) => Mechanical equation

fundamental object... particle (kinetic approach)



We can distinguish each particle from others.

Focus on the **Position** and **Velocity** of **every particle:** $\boldsymbol{x}_i(t),\ \boldsymbol{v}_i(t)$

Solve **mechanical equation** with one independent variable **(time)** for all particles.

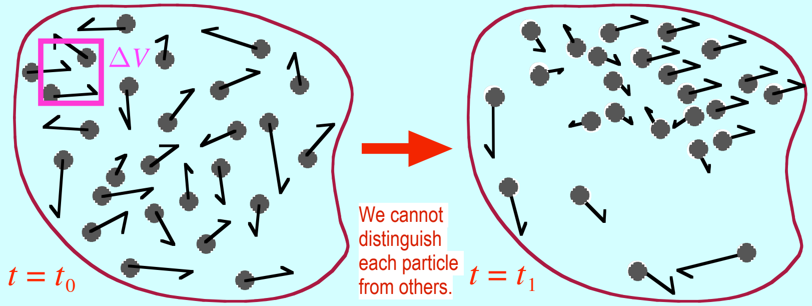
$$\begin{cases} \dfrac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i \\ m_i \dfrac{d\boldsymbol{v}_i}{dt} = \boldsymbol{F}_i \end{cases} \quad i = 1, 2, 3, \ldots, N$$

(Determine the **evolutionary path** of **every particle** => the most complete solution)

II. Intermediate N system (discrete system*) => Boltzmann's equation

*Sufficient number of particles **exist in each local** $\triangle V$ **to determine the distribution of particles** there.

fundamental object... particle (kinetic approach)



Focus on the **Distribution** of **particles:** $f(\boldsymbol{x}, \boldsymbol{v}, t)$

(Give up deriving the evolution of every particle => focus on the **evolution** of **particle distribution**)

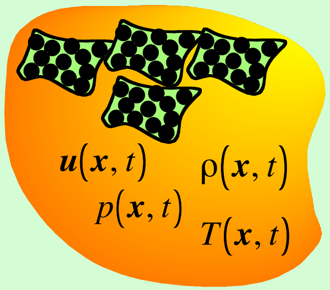
Solve **Boltzmann's equation** with seven independent variables **(position, velocity, time)** for distribution function.

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \bullet \frac{\partial f}{\partial \boldsymbol{x}} + \frac{\boldsymbol{F}}{m} \bullet \frac{\partial f}{\partial \boldsymbol{v}} = \left(\frac{\delta f}{\delta t}\right)_C$$

III. Large N system (continuous system*) => Fluid dynamics equations

*Sufficient number of particles **exist in each fluid element** and **keep staying in** it.

fundamental object... fluid element (fluid approach)



Use **continuum approximation:** fluid elements fill up the entire volume of system => particle-based field: $\rho, \boldsymbol{u}, P, T$

Focus on the **thermal & dynamical evolution** of **every fluid element**: $\rho(\boldsymbol{x}, t)$, $\boldsymbol{u}(\boldsymbol{x}, t)$, $T(\boldsymbol{x}, t)$, $p(\boldsymbol{x}, t)$

Take the **average of Boltzmann's equation** using **Maxwellian distribution function** to derive **fluid dynamics equations.**

Solve **fluid dynamics equations** with four independent variables **(position, time)** for fluid elements.

Additional comments on plasma state, kinetic & fluid approaches

Plasma state... *4th state of a system* in which

there are **many charged particles**

these particles **behave collectively** => *local charge neutrality*

Kinetic approach... *more fundamental approach* in the sense that

it is based on **real object** (**particle**)

it can be used **even for non-plasma state** (*each particle behaves independently*)

Fluid approach... *less fundamental approach* in the sense that

it is based on **virtual object** (**fluid element**)

it can be used **only for thermal state** (*velocity distribution of particles is given by Maxwellian distribution*)